CONTOURLET DOMAIN IMAGE DENOISING USING NORMAL INVERSE GAUSSIAN DISTRIBUTION

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ABSTRACT

A new contourlet-based method is introduced for reducing noise in images corrupted by additive white Gaussian noise. It is shown that a symmetric normal inverse Gaussian distribution is more suitable for modeling the contourlet coefficients than formerly-used generalized Gaussian distribution. To estimate the noise-free coefficients, a Bayesian maximum *a posteriori* estimator is developed utilizing the proposed distribution. In order to estimate the parameters of the distribution, a moment-based technique is used. The performance of the proposed method is studied using typical noise-free images corrupted with simulated noise and compared with that of the other state-of-the-art methods. It is shown that compared with other denoising techniques, the proposed method gives higher values of the peak signal-to-noise ratio and provides images of good visual quality.

Index Terms— Contourlet transform, image denoising, normal inverse Gaussian distribution, maximum a posterior estimator.

1. INTRODUCTION

Image denoising is an important image processing problem, which is realized by reducing noise from an image and preserving its features. Noise removal problem using multiscale transforms have been investigated in many recent works [1]-[4]. The contourlet transform-based approaches have led to a significant success in image denoising as compared to former wavelet-based methods [5], [6]. Usually, the thresholding of the contourlet coefficients is performed using a simple soft or hard thresholding function. However, it has been shown that a shrinkage function using Bayesian approach can provide noise reduction performance superior to that of the thresholding schemes [1], [2]. Such a shrinkage function can be developed based on the assumption of mutually independent coefficients by minimizing a Bayesian risk under the maximum *a posteriori* (MAP) criterion. In this method, the contourlet transforms are modelled by a particular probability density function (PDF). The performance of the Bayesian MAP estimator depends on the correctness of the contourlet coefficients prior. The objective of this paper is to introduce a new MAP-based image denoising method in the contourlet domain using the normal inverse Gaussian distribution as a prior for contourlet coefficients. It is known that the contourlet subband coefficients of an image have significantly non-Gaussian and heavy-tailed properties that are best described by heavy-tailed distributions such as formerly used generalized Gaussian distribution [7], [8]. In this work, we propose the global modeling of the contourlet coefficients of an image by the normal inverse Gaussian (NIG) PDF [9]-[11]. Thus, the MAP estimator based on NIG model is developed and a modified shrinkage function corresponding to this estimator is obtained. A moment-based method is used for estimating the parameters of the NIG distribution from the noisy coefficients.

The paper is organized as follows. In Section 2, the proposed NIG model for the contourlet coefficients is presented. The image denoising algorithm using MAP-based estimator is described in Section 3. Experimental results are presented in Section 4. Section 5 concludes the paper.

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2. PROPOSED MODELING

In view of the fact that the contourlet coefficients of an image are non-Gaussian [6]-[8], i.e., having large peaks around zero and tails heavier than that of a Gaussian PDF, a proper distribution to model the statistics of the contourlet coefficients would be a heavy-tailed PDF. It has been shown in [6] that the generalized Gaussian (GG) distribution can model the contourlet coefficients. In this work, we propose using the normal inverse Gaussian (NIG) distribution to model the contourlet coefficients of an image as an alternative to the GG distribution. The NIG distribution is a variance mean-mixture of a Gaussian PDF with an inverse Gaussian PDF [9], [10]. The density function of a random variable $X \sim NIG(\alpha, \beta, \mu, \delta)$

is given by

$$P_{NIG}(x) = \frac{\alpha\delta e^{\delta\gamma+\beta(x-\mu)}}{\pi} \frac{K_1\alpha\sqrt{\delta^2+(x-\mu)^2}}{\sqrt{\delta^2+(x-\mu)^2}} \quad (1)$$

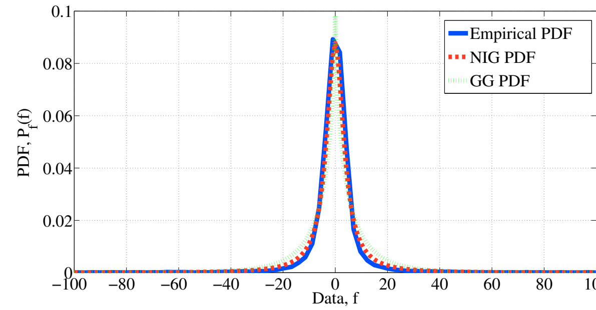
where K_1 is the modified Bessel function of the second kind with index 1 and $\gamma = (\alpha^2 - \beta^2)^{1/2}$. The shape of the NIG distribution is specified by four parameters, namely, α, β, μ and δ, which are shape, skewness, location, and scale parameters, respectively. The parameters are bound as $0 \leq |\beta| < \alpha$, $\delta > 0$ and $-\infty < \mu < \infty$. For zero-mean and symmetric data distribution $\mu = \beta = 0$. The PDF of the empirical data for the two finest subbands of the *Barbara* image as well as that of the GG and NIG distributions are shown in Fig. 1. From this figure, it is seen that the NIG prior provides a better fit to the empirical distribution than that achieved by the GG distribution. The Kolmogorov-Smirnov (KS) statistic given by $max|\int P_f(f) - \hat{P}_f(f)df|$, in which $P_f(f)$ denotes the PDF of the random variable and $\hat{P}_f(f)$ represents the PDF of the empirical data, is also used to quantify the closeness of the empirical data to an assumed prior for the contourlet coefficients. The value of KS statistic is found to be 0.0926 for the NIG and 0.1358 for the GG distribution, indicating that the NIG distribution fits the empirical data more closely than the GG distribution does. Similar results are also observed for other test images. In order to estimate the NIG parameters, we use its moment generating function $M(t) = E[e^{tx}]$, which is given by

$$M_{NIG}(x) = \exp\left(\mu t + \delta\left(\sqrt{\alpha^2+\beta^2} - \sqrt{\alpha^2-(\beta+t)^2}\right)\right) \quad (2)$$

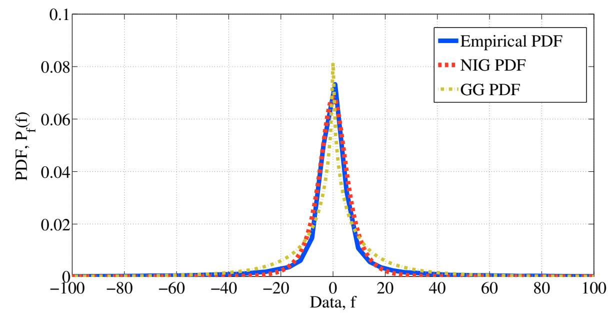
The first four moments of the random variable X are given by

$$\begin{aligned} E(X) &= \mu + \frac{\delta\beta}{\gamma} \\ V(X) &= \frac{\delta\alpha^2}{\gamma^3} \\ S(X) &= \frac{3\beta}{\alpha(\delta\gamma)^{1/2}} \\ K(X) &= \frac{3(1+4(\frac{\beta}{\alpha})^2)}{\delta\gamma} \end{aligned} \quad (3)$$

Then, the parameters of the NIG distribution can be estimated through inserting the sample moments $\overline{m}_i, i = 1, 2, \ldots$, into



(a)



(b)

Fig. 1: PDFs of empirical data (solid) as well as the GG (dotted) and NIG (dashed) distributions for the two finest subbands of the *Barbara* image.

(3) and solving for each parameter as

$$\begin{aligned} \hat{\gamma} &= \frac{3}{\overline{m}_2(3\overline{m}_4 - 5\overline{m}_3^2)^{1/2}} \\ \hat{\beta} &= \frac{\overline{m}_2\overline{m}_3\hat{\gamma}^2}{3} \\ \hat{\mu} &= \overline{m}_2 - \frac{\hat{\beta}\hat{\delta}}{\hat{\gamma}} \\ \hat{\delta} &= \frac{\overline{m}_2\hat{\gamma}^3}{(\hat{\beta}^2+\hat{\gamma}^2)} \end{aligned} \quad (4)$$

3. BAYESIAN MAP ESTIMATOR

Suppose that a noisy image is decomposed to $j = 1, ...J$ scales and $d = 1, ..., D$ direction subbands by the contourlet transform. Then, we have $y_j^d(m,n) = x_j^d(m,n) + \eta_j^d(m,n)$, where η is the noise term which is supposed to be reduced. In order to estimate the noise-free coefficients x in contourlet domain, a Bayesian MAP estimator is developed through modeling the contourlet coefficients of a noisy image by the NIG PDF. The Bayesian MAP estimator of x, given noisy observation y, can be derived as

$$\hat{x}(y) = argmaxP_{x|y}(x|y) \quad (5)$$

According to the Bayesian rule, (5) can be rewritten as

$$\hat{x}(y) = argmax P_{y|x}(y|x) \tag{6}$$

where $P_x(x)$ is the PDF of the contourlet coefficients of a noise-free image. Then, (6) can be rewritten as

$$\hat{x}(y) = argmax P_{\eta}(y - x)P_x(x) \tag{7}$$

where $P_{\eta}(\eta)$ is the noise PDF. In the proposed denoising method, the noise is assumed to be white Gaussian with a zero mean and a standard deviation of σ_{η}. If σ_{η} is unknown, it may be estimated by applying the robust median absolute deviation method [12] in the finest subband of the observed noisy coefficients. The PDF of noise is given by

$$P_{\eta}(\eta) = \frac{1}{\sqrt{2\pi\sigma_{\eta}^2}} \exp(-\frac{\eta^2}{2\sigma_{\eta}^2}) \tag{8}$$

To obtain the MAP estimate, after inserting (8) into (7), the derivative of the logarithm of the argument in (7) is set to zero resulting in

$$\frac{\hat{x} - y}{\sigma_{\eta}^2} + \frac{\partial}{\partial x}(-ln(P_x(x))) = 0 \tag{9}$$

To ensure the consistency of the signs of $\hat{x}$ and y, an approximate bounded solution of (9) is obtained as [13]

$$\hat{x}(y) = sign(y)(|y| - \sigma_{\eta}^2|M|)_+ \tag{10}$$

where $(z)_+ = max(z, 0)$ and

$$M = -\beta + \frac{2(y-\mu)}{\sigma^2 + (y-\mu)^2} + \frac{\alpha(y-\mu)}{\sqrt{\delta^2 + (y-\mu)^2}} \frac{K_0(\alpha\sqrt{\delta^2 + (y-\mu)^2})}{K_1(\alpha\sqrt{\delta^2 + (y-\mu)^2})} \tag{11}$$

Thus, the denoising method can be summarized as follows

1. Apply the contourlet transform on the noisy image and obtain the contourlet coefficients.

2. Estimate the parameters of the NIG distribution from the noisy coefficients by using (3) and (4).

3. Estimate the noise-free coefficients of all detail subbands using the Bayesian MAP estimator in (10).

4. Apply the inverse contourlet transform on the estimated noise-free coefficients to obtain the denoised image.

Table 1: PSNR VALUES OBTAINED USING DIFFERENT DENOISING METHODS FOR TWO OF THE TEST IMAGES, THE *BARBARA* AND *PEPPERS* IMAGES

	Standard deviation			
Method	10	15	20	25
Barbara				
Noisy image	28.13	24.61	22.11	20.17
Proposed	**31.23**	29.51	**28.37**	**27.29**
Adaptive-shrink	-	**29.96**	28.36	27.23
SURE-shrink	28.20	24.64	22.13	20.19
Bayes-shrink	30.28	27.46	26.09	25.56
Adaptive wiener filter	28.31	27.37	26.44	25.21
Visu-shrink(soft)	27.34	24.44	22.19	20.06
Visu-shrink(hard)	28.78	26.85	25.46	24.46
Peppers				
Noisy image	28.13	24.61	22.11	20.17
Proposed	**32.56**	**31.11**	**29.30**	**27.41**
Adaptive-shrink	31.95	30.01	28.37	27.23
SURE-shrink	28.20	24.65	22.13	20.18
Bayes-shrink	30.97	29.63	28.94	26.85
Adaptive wiener filter	31.80	29.56	28.41	27.23
Visu-shrink(soft)	29.70	27.88	25.31	23.20
Visu-shrink(hard)	29.34	28.12	27.26	26.50

4. SIMULATION RESULTS

The performance of the proposed method is verified by conducting experiments on the standard test images and comparing the results to that obtained by using some of the existing methods, namely, adaptive wiener filter, Visu-shrink (soft and hard), adaptive-shrink [1], Bayes-shrink [2] and SURE-shrink [14]. The experiments are performed on images corrupted with various levels of Gaussian noise, specifically σ_{η} varying from 10 to 25. The noisy image is transformed by the contourlet transform with three levels of decomposition and 8, 8 and 4 directions from finer to coarser scales, respectively. Since the contourlet transform is a shift-variant transform, in the proposed contourlet domain denoising, in order to overcome the possible Pseudo-Gibbs phenomena in the neighborhood of discontinuities, the cycle spinning method [15] is performed on the observed data. The peak signal-to-noise ratio (PSNR) is used as a subjective performance criterion. Table I gives the values of PSNR for some of the existing methods for two of the test images, *Barbara* and *Peppers*. It can be seen from this table that the proposed method generally yields higher PSNR values for a given range of noise standard deviations. The denoised *Barbara* image obtained from various methods with $\sigma_{\eta} = 10$ is illustrated in Fig. 2. It can be seen from this figure that the proposed denoising method generally provides better visual quality than some of the existing methods.

5. CONCLUSION

In this work, we have proposed a new image denoising method in the contourlet domain. The proposed method has been obtained by modeling the contourlet coefficients using the normal inverse Gaussian distribution. It has been shown that this distribution can model the contourlet subband coefficients more accurately than formerly-used generalized Gaussian distribution can. The noisy image has been decomposed into various scales and directional subbands via contourlet transform. The noise in all detail subbands has been removed by a closed-form Bayesian MAP estimator using the NIG prior. In order to estimate the parameters of the assumed distribution, a moment-based method has been used. Experiments have been carried out to compare the performance of the proposed method with that provided by some of the existing methods. The simulation results have shown that the proposed denoising method outperforms some of the existing methods in terms of the PSNR values and provides superior visual quality denoised images.

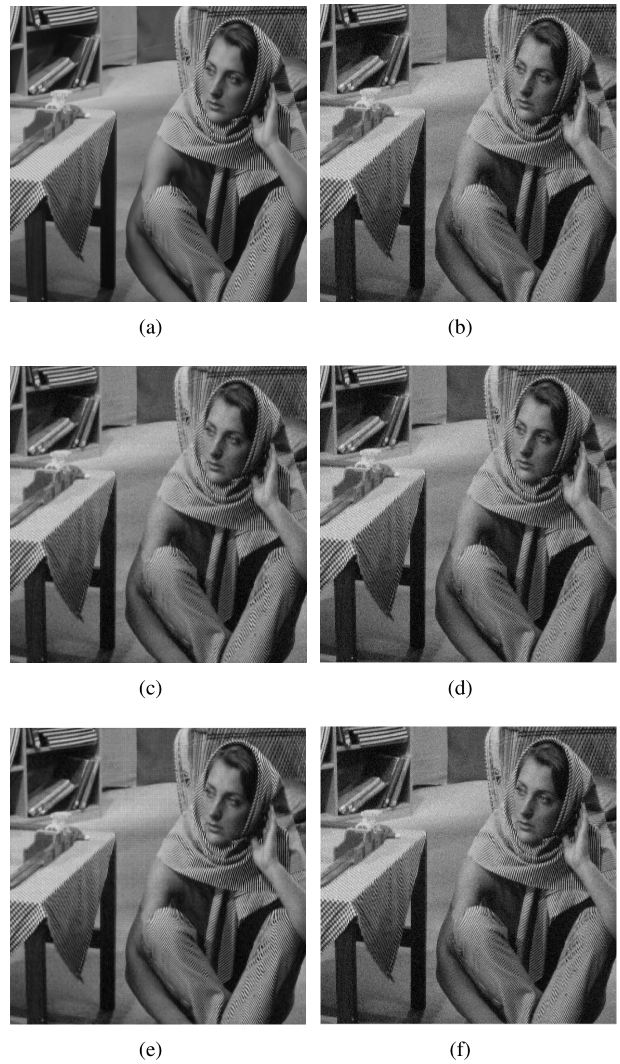


Fig. 2: Visual comparison of various denoising methods with $\sigma_\eta = 10$. (a) Original *Barbara* image. (b) Noisy image, PSNR= 28.13. (c) Soft thresholding PSNR=27.34. (d) Hard thresholding, PSNR=28.78. (e) Bayes-shrink, PSNR= 30.28. (f) Proposed, PSNR= 31.23.

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