ADAPTIVE DIMENSIONALITY REDUCTION METHOD USING GRAPH-BASED SPECTRAL DECOMPOSITION FOR MOTOR IMAGERY-BASED BRAIN-COMPUTER INTERFACES

Golnar Kalantar †*, Hamidreza Sadreazami* ∓*, Arash Mohammadi* † *and Amir Asif* ∓

†Concordia Institute for Information System Engineering (CIISE)
∓Department of Electrical and Computer Engineering (ECE)
Concordia University, Montreal, Quebec, Canada H3G 1M8

ABSTRACT

In this work, we consider the problem of electroencephalography (EEG) signal classification for motor imagery brain-computer interfaces. The goal is to identify the pattern of the brain activity using a robust method for pre-processing, processing, and classification of the EEG signals. To this end, a new graph-based framework is proposed to reduce the dimensionality of the data by taking into account not only the geometrical structure of the channels/electrodes, but also the correlation between the EEG signals. The most significant feature vectors required for EEG signals classification are adaptively selected through spectral decomposition of the data using the graph Laplacian matrix. The tangent space mapping method is then applied to bring the captured data into Euclidean space. In order to classify the dimensionally-reduced EEG signals, the linear support vector machine algorithm is employed. Experiments are conducted on five different subjects consisting of right hand and right foot motor imagery actions. The results show that the proposed method can provide higher classification accuracy as compared to the other existing methods that we tested.

***Index Terms*—** Brain-computer interfaces, Euclidean space, principal component analysis, tangent space mapping, graph signal processing.

1. INTRODUCTION

Brain activity recording has been a trending research field resulting in broad range of technological developments, from commercial electroencephalogram (EEG) headsets to rehabilitation robots [1] helping paralyzed patients move their muscles by merely thinking about the movement. Motor imagery (MI) [2, 3] is the mental execution of a movement without any real movement or peripheral (muscle) activation. The variation in brain activity is quantified from Electrophysiological recording such as EEG during the MI task. Patients receive visual or kinesthetic feedback in order to promote the brain response to the MI task. For an efficient and patient-friendly neuro-rehabilitation with Brain-Computer Interfaces (BCIs) [4–6], it is crucial to establish highly accurate technologies for the interpretation of brain activities [7]. A BCI system is a real-time communication system designed to provide users with a mean of communicating commands independent of the connectivity between brain's normal output channels and peripheral nerves and muscles [12]. Among several non-invasive methods of tracking and recording brain wave patterns, EEG has been widely used to capture brainwaves, i.e., the electric field generated by the central nervous system. This popularity is due to EEG's simplicity, inexpensiveness, and high temporal resolution [9]. It is known that the EEG is capable of detecting changes in brain's electrical activities on a millisecond-level, which is one of the few non-invasive available techniques with such a high temporal resolution.

On the other hand, unprocessed EEG signals are known to have a poor spatial resolution owing to volume conduction [10]. Since analyzing spatial features of the activities is decisive in several situations, a multichannel measurement of EEG, where electrodes are installed at various locations on a human head, has been widely used. A rather blurred image of the brain activity often results from multichannel EEG signals, because of low signal-to-noise ratio (SNR) [11]. EEG signals are produced by excitement of millions of neurons near each electrode. Consequently, the signal recorded at the electrodes that are located near each other are similar [13]. In view of this, various methods have been employed to reduce the dimensionality of the EEG signals. These include, for instance, principal component analysis (PCA), Wiener filtering, common spatial patterns (CSP), and independent component analysis (ICA) based techniques, to name a few. These techniques are based on decomposing the raw EEG data into spatial patterns and maximizing the variance between the resulting populations [21]. In [23], PCA has been applied to reduce the dimensionality based only on the observations values. A dimensionality reduction technique for EEG data has been proposed in [7] by describing the geometric structure of the graph formed by the electrodes and spatially filtering the data with this static graph of EEG signals. In [8], an iterative method has been proposed for electrode selection in BCI experiments using the Riemannian distance between spatial covariance matrices.

Recently, graph signal processing (GSP) [16–18, 22, 26] has provided a new framework for representing model relations among data samples. For data-oriented applications, a weighted graph can be identified to capture similarities within data samples. For instance, an image may be represented by associating image pixels with graph nodes [19]. The corresponding graph can be analyzed using newly-defined GSP techniques [16]. In brain imaging, it is now possible to non-invasively infer the anatomical connectivity of distinct functional regions of the cerebral cortex, and this connectivity may be represented by a weighted graph with the vertices corresponding to the functional regions of interest [18].

In view of this, in this work, we propose a new graph-based classification method for motor imagery brain computer interface, referred to as the GC-BCI. The proposed GC-BCI framework is realized by using a novel dimensionality reduction technique which takes into account not only the geometrical structure of the electrode channels, but also the correlation coefficients obtained from

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the EEG signals. Accordingly, the graph Fourier transform is obtained as a spectral decomposition of the graph which is later applied to the EEG signals for dimensionality reduction purpose. A tangent space mapping technique is employed to project data from the Riemannian to the Euclidean domain. The PCA method is applied for selecting the most significant features for classification purpose. Finally, the linear support vector machine is used to solve a two-class classification problem.

The paper is organized as follows. Section 2 formulates the problem and presents the proposed GC-BCI framework. Simulation results are provided in Section 3. Finally, Section 4 concludes the paper.

2. THE PROPOSED GC-BCI FRAMEWORK

Throughout the paper, the following notation is used: non-bold letter x denotes a scalar variable, lowercase bold letter $\boldsymbol{x}$ represents a vector, and capital bold letter $\boldsymbol{X}$ denotes a matrix. The real domain is represented by $\mathbb{R}$. The transpose of a matrix $\boldsymbol{X}$ is denoted by $\boldsymbol{X}^T$.

We consider supervised learning from EEG signals based on the available set of EEG epochs (trials) denoted by $\boldsymbol{X}_i \in \mathbb{R}^{N_{\text{ch}} \times N_{\text{t}}}$, for $(1 \leq i \leq N_{\text{Trial}})$, where N_{Trial} is the total number of trials used for processing; N_{ch} is the number of EEG channels (electrodes), and; N_{t} is the number of time samples collected from each electrode in one trial. The training dataset is denoted by $\{(\boldsymbol{X}_i, l_i)\}$, for $(1 \leq i \leq N_{\text{Trial}})$, where l_i represents the label corresponding to the ith trial, e.g., l_i could be "right foot" or "right hand". Parameters rTR and rTS are the number of eigenvectors required dimensionality reduction process of training and test sets, respectively. For vector $\boldsymbol{X}_i$, the sample covariance matrix is defined as

$$\boldsymbol{C}_i = \frac{1}{N_{\text{t}} - 1} \big(\boldsymbol{X}_i - \boldsymbol{\mu}_i\big)\big(\boldsymbol{X}_i - \boldsymbol{\mu}_i\big)^T, \tag{1}$$

where $\boldsymbol{\mu}_i$ is the column-wise mean of $\boldsymbol{X}_i$. The proposed GC-BCI framework consists of the following main tasks: (i) Pre-processing; (ii) Dimensionality reduction; (iii) Mapping to tangent space; (iv) Pre-classification; and (v) Classification. Below and in each subsection, the aforementioned tasks are described in details.

2.1. Pre-Processing

Before processing EEG signals for classifying MI tasks, a pre-processing step is typically required. At this stage, initially the power line interference is removed by applying a notch filter. Then, bandpass filtering is applied to extract specific frequency contents of the signal. This step is then followed by downsampling. The signal used for processing is extracted from specific period of each trial time interval. This step is conventionally done by selecting a predefined time interval after a visual cue and selecting of one sample value out of n samples.

In order to take into consideration most of subject's response to each stimulus, we propose the use of other methods for data smoothing prior to the downsampling step, namely, simple averaging (SA), simple moving average (SMA), weighted moving average (WMA) and moving median (MM). More specifically, unlike [7], instead of randomly choosing one value, i.e., random selection (RS) within the time interval between two consecutive visual cues, we compute the average values using SA, SMA and WMA, or the median value using MM. For all the methods, the window size is set to 10 samples at a time. In order to investigate the effectiveness of the proposed data smoothening methods, we obtain the reconstruction error resulted from each method after reconstructing the signal.

Table 1. Average reconstruction error obtained using various data smoothening methods for training datasets.

	FF: rTR = 41	EV: 80%	PRD: 2%
	AA		
RS	4.32	0.90	69
SA	4.21	0.87	68
SMA	4.21	0.86	68
WMA	4.08	0.83	67
MM	4.33	0.91	69
	AW		
RS	3.39	0.68	59
SA	3.27	0.66	57
SMA	3.27	0.65	57
WMA	3.12	0.62	55
MM	3.39	0.69	59
	AL		
RS	3.63	0.68	58
SA	3.51	0.66	57
SMA	3.50	0.63	57
WMA	3.37	0.62	55
MM	3.62	0.69	58
	AY		
RS	4.20	0.68	64
SA	4.06	0.65	63
SMA	4.06	0.65	63
WMA	3.90	0.62	62
MM	3.19	0.69	64
	AV		
RS	4.39	0.83	67
SA	4.34	0.81	66
SMA	4.34	0.81	66
WMA	3.27	0.79	65
MM	3.41	0.85	67

Table 1 lists the reconstruction error obtained using training datasets of different subjects. The signal reconstruction is performed using a fixed number of features (FF), percentage root-mean-square difference (PRD) [24] and Explained Variance (EV) [25]. For the FF method, similar to [7], we consider 41 principal components for dimension reduction, for EV method, a 80% cut-off variance explained by the eigenvalues are considered and the corresponding PRD value is obtained, and in the last column of the Table 1, we compute the number of eigenvectors required for a 2% reconstruction error according to the PRD method. It is seen from this table that WMA provides lower reconstruction error when smoothening the EEG signals. Henceforth, we employ this filter in the pre-processing step of our proposed method. In the next subsection, the proposed graph-based dimensionality reduction method is presented.

2.2. Graph-based Dimensionality Reduction

Graph signal processing is an emerging field that offers a framework for applying classical signal processing to signals defined on graphs [16]. A weighted graph is a triplet $G = (V, E, K)$ consisting of a finite set V of vertices (electrode channels) and a finite set E of edges with the corresponding weights $[k_{pq}]_{n \times n} \in \boldsymbol{K}$. The weights k_{pq} can be defined as a function of proximity between vertices (electrodes) p and q, as given by

$$\boldsymbol{K}_{\text{PG}} = \exp\left(-\frac{\boldsymbol{D}(p,q)^2}{2\sigma_d^2}\right), \tag{2}$$

where p and q are the electrode positions, and $\boldsymbol{D}(p,q)$ denotes the distance between the two electrodes. In this work, in order to take into account the dependencies of the data captured at each electrode, we propose a new weight matrix which is a function of both the electrode proximity and correlation coefficients obtained from the EEG signals.

$$\boldsymbol{K}_{\text{VPG}} = \exp\left(-\frac{\boldsymbol{D}(p,q)^2}{2\sigma_d^2}\right).\exp\left(-\frac{(1-\|\boldsymbol{\rho}(p,q)\|)^2}{2\sigma_\rho^2}\right), \tag{3}$$

where σ_d and σ_ρ specify the amount of exponential decay rate, and

$$\boldsymbol{\rho}(p,q) = \frac{c_{pq}}{\sqrt{c_{pp}c_{qq}}}, \tag{4}$$

obtained using the elements of the covariance matrix $\boldsymbol{C}$, given in (1). Accordingly, the degree matrix $\boldsymbol{Di}$ is defined using the weight matrix as

$$\boldsymbol{Di} = \text{diag}\left\{\sum_q k(1,q),...,\sum_q k(n,q)\right\}. \tag{5}$$

The graph Laplacian matrix is derived from $\boldsymbol{K}$ and plays an important role in describing the underlying structure of the graph signal. The graph Laplacian and its normalized version are defined as $\boldsymbol{L} = \boldsymbol{Di} - \boldsymbol{K}$ and $\boldsymbol{L}_{\text{normal}} = \boldsymbol{I} - \boldsymbol{Di}^{-1/2}\boldsymbol{K}\boldsymbol{Di}^{-1/2}$, where $\boldsymbol{I}$ is the identity matrix. Spectral graph theory studies the graph properties in terms of eigenvalues and eigenvectors associated with the Laplacian matrix of the graph. The set of eigenvectors of $\boldsymbol{L}_{\text{normal}}$ constitute the basis function for the underlying signal defined on graph, and its eigenvalues are known as the corresponding graph frequencies. The eigen decomposition of the real and symmetric normalized Laplacian is given by

$$\boldsymbol{L}_{\text{normal}} = \sum_i \boldsymbol{\lambda}_i \boldsymbol{u}_i \boldsymbol{u}_i^T, \tag{6}$$

where $\{\boldsymbol{\lambda}_i\}_{i=1,...,n}$ is the set of eigenvalues and $\{\boldsymbol{u}_i\}$ the set of orthogonal eigenvectors used for dimension reduction.

Let $\boldsymbol{U}$ contain the $\boldsymbol{L}$'s first r eigenvectors corresponding to the first r eigenvalues of $\boldsymbol{L}$ sorted in ascending order. The proposed dimensionality reduction technique based on graph spectral theory employes matrix $\boldsymbol{U}$ to represent the EEG signals with lower number of features $\mathbf{F}_r$ as given by

$$\mathbf{F}_r = \boldsymbol{U}_r^T \boldsymbol{X}. \tag{7}$$

It should be noted that the first r eigenvectors correspond to the first r low-frequency basis functions in graph spectral domain. It should be noted that r can be adaptively determined for different subjects using the PRD method.

The dimension reduction step is followed by mapping the data from the existing manifold to the Euclidean space. To this end, tangent space mapping method is employed as a bridge operation to enable us to treat the data transferred to Euclidean space as vectors. This mapping method is discussed in the next subsection.

Table 2. Classification accuracy performance for predicting two classes and the corresponding standard deviation (std), obtained using the proposed graph-based dimensionality reduction methods, namely, PG and VPG.

Subject AA (168 Train+ 112 Test)			
PRD<2.5% rTR=rTS=60		PRD<4.15% rTR=41, rTS=43	
VPG	PG	VPG	PG
82.90± 1.23	80.99± 1.64	76.29± 1.61	74.38± 2.14
Subject AL (224 Train+ 56 Test)			
PRD<2% rTR=59, rTS=56		PRD<3.5% rTR=41, rTS=39	
VPG	PG	VPG	PG
97.73± 0.30	97.62± 0.23	97.62± 0.27	97.38± 0.31
Subject AW (56 Train+ 224 Test)			
PRD<2% rTR=rTS=58		PRD<3.3% rTR=rTS=41	
VPG	PG	VPG	PG
92.58± 2.82	91.30± 2.75	93.70± 2.21	90.83± 3.14
Subject AV (84 Train+ 196 Test)			
PRD<2% rTR=rTS=66		PRD<4.29% rTR=41, rTS=42	
VPG	PG	VPG	PG
67.79± 2.97	66.87± 2.69	66.99± 2.83	64.21± 3.32
Subject AY (28 Train+ 252 Test)			
PRD<2% rTR=rTS=66		PRD<4% rTR=41, rTS=38	
VPG	PG	VPG	PG
83.14± 5.76	82.68± 5.67	80.29± 4.64	84.32±4.61

2.3. Tangent Space Mapping

It is known that the sample covariance matrices belong to the Riemannian manifold of the symmetric and positive definite matrices. However, several significant and commonly used state-of-the-art methods of machine learning, and specifically, classification techniques, are mostly designed to be applied to datasets in the Euclidean space. In view of this, we employ the tangent space mapping technique [20] to project data to Euclidean space as vectors.

Let a Riemannian manifold $\mathcal{S}(n)$ be a space of $n \times n$ symmetric positive definite matrices given by $\mathcal{S}(n) = \{\boldsymbol{S} \in \mathcal{M}(n), \boldsymbol{S}^T = \boldsymbol{S}\}$, where $\mathcal{M}(n)$ is the space of all square real matrices. The set of all the matrices is denoted as $\mathcal{C}(n) = \{\boldsymbol{C} \in \mathcal{S}(n), \boldsymbol{u}^T\boldsymbol{C}\boldsymbol{u} > 0\}$, which is not Euclidean. Tangent space mapping provides a Euclidean tangent space, $\boldsymbol{T}_Q\boldsymbol{C}(n)$ at the point Q, which approximates the aforementioned Riemannian manifold through the following steps:

- Compute the set of sample covariance matrices for each trial as given in (1).
- Compute the mean Riemannian distance as

$$\bar{\boldsymbol{C}} = \frac{1}{N_{\text{Trial}}} \sum_{i=1}^{N_{\text{Trial}}} \boldsymbol{C}_i. \tag{8}$$

Table 3. Performance comparison of the proposed graph-based method in two-class classification problem with that provided by [7] and [8].

	PG		VPG		[7]	[8]
	PRD: rTR = 41	PRD: 2%	PRD: rTR = 41	PRD 2%	rTR = 41	rTR = 10
AA	74.38± 2.14	80.99± 1.64	76.29± 1.61	82.90± 1.23	81.43± 10.9	74.1
AL	97.38± 0.31	97.62± 0.23	97.62± 0.27	97.37± 0.30	97.50± 2.98	98.2
AW	90.83± 3.14	91.30± 2.75	93.70± 2.21	92.58± 2.82	98.57± 0.79	77.7
AV	64.21± 3.32	66.87± 2.69	66.99± 2.83	65.79± 2.97	69.29± 5.56	59.2
AY	84.32± 4.61	68.82± 5.67	80.29± 4.64	83.14± 5.76	93.93± 4.30	80.6
Average	82.22± 2.70	81.12± 2.60	82.98± 2.31	84.35± 2.69	88.14± 4.90	78

- Compute the map $\mathbf{s}_i$ from $\boldsymbol{C}$ to $\boldsymbol{T}_Q\boldsymbol{C}(n)$ as

$$\mathbf{s}_i = \text{Upper}\left(\log\left(\bar{\boldsymbol{C}}^{\frac{-1}{2}}\boldsymbol{C}_i\bar{\boldsymbol{C}}^{\frac{-1}{2}}\right)\right), \tag{9}$$

where Upper is used to weigh the upper triangular half of a matrix and vectorize it. In particular, it assigns 1 as the weight for main diagonal and $\sqrt{2}$ for off-diagonal entries. The resulting feature vectors are further trimmed and the most relevant ones are selected for classification purpose. This pre-classification step is presented in the next subsection.

2.4. Pre-classification

It is known that the high dimensional features may lead to poor classification performance. This is due to the fact that large number of irrelevant features not only degrades the generalization of the model, but also imposes computational cost. In view of this and in order to determine the most significant feature vectors of $\mathbf{s}_i$ in the tangent space mapping process, which are maximally related to the desired classes, we use a PCA-based feature selection method. To this end, similar to [7], the first 10 principal components are selected and fed into the classifier as its input. In addition, for PRD = 2%, eigenvectors are adaptively selected for each subject. It should be noted that using the PRD method, one can adaptively compact feature vectors for a better classification result.

2.5. Classification

In order to classify the selected feature vectors as representative of right hand or right foot MIs, we employ the linear support vector machine (SVM) algorithm. The SVM uses training feature vectors to learn a decision boundary that separates these two classes by projecting data into a higher dimensional space using a kernel function. Once the decision boundary is learned, the SVM determines the class membership of a newly-observed feature vector according to the side of boundary that the vector falls.

3. RESULTS

The proposed method is benchmarked on the dataset IVa from the BCI competition III taken from http://www.bbci.de/competition/iii/. The EEG positioning is based on $10-20$ standard system. The dataset is composed of EEG recordings of 118 electrodes. The experiment is a classical cue-based MI paradigm in which each of 5 subjects, namely, AA, AL, AV, AW, and AY, perform 280 trials of right hand and right foot MIs. In the pre-processing step, the EEG signals are bandpass filtered in the frequency band $[8-30]$ Hz, including alpha and beta bands, by a 5th order *Butterworth* filter. The time interval is restricted to the segment located from 0.5s to 4s after the cue. The WMA filter is then applied to smoothen the data.

In order to obtain the most significant features for each subject, we propose the use of PRD to adaptively determine the required number of eigenvectors from which data can be reconstructed with a predefined error. The corresponding size of datasets for different subjects are given in Table 2.

Table 2 gives the classification accuracy and its corresponding standard deviation averaged over 400 runs, obtained using the proposed method using physical graph (PG) and value-physical graph (VPG). It is seen from this table that proposed method using VPG outperforms its PG counterpart by almost 10%. This is due to the fact that the VPG is built using the electrode channel proximity (PG case) as well as the correlation coefficients of the EEG signals.

We now compare the performance of the proposed GC-BCI method to that obtained from the other existing methods, in terms of the classification accuracy. Table 3 gives the comparison results of the proposed method using VPG and PG and that provided by [7] and [8], when constant or adaptive number of features are selected. It is seen from this table that the proposed method provides higher classification accuracy for various subjects as compared to those yielded by [7] and [8]. In addition, the standard deviation of the classification accuracy obtained using the proposed method is lower than that provided by [7].

4. CONCLUSION

In this paper, we have proposed a new dimensionality reduction technique for classifying EEG signals obtained from motor imagery brain-computer interface systems. The proposed GC-BCI method has been established by leveraging the recent advances in the field of graph signal processing. The proposed method is composed of an efficient graph-based dimensionality reduction technique followed by tangent space mapping of the EEG signals to the Euclidean space and a pre-classification step using the PCA-based feature selection method. Experiments have been conducted on a set of EEG signals obtained from the BCI competition. The results have shown that the proposed method produces encouraging results providing high recognition accuracy for two-class classification.

As future works, the obtained results will be further improved by employing graph-based smoothening filters and a more accurate manifold mapping. The proposed method will also be examined for a multi-class classification problem.

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