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Color Image Denoising using Multivariate Cauchy PDF in the Contourlet Domain

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***Abstract*—A new color image denoising method in the contourlet domain is proposed for reducing noise in images corrupted by Gaussian noise. This method takes into account the statistical dependencies among the contourlet coefficients of the RGB color channels. To this end, the multivariate Cauchy distribution is employed to capture these inter-channel dependencies. This model is then exploited in a Bayesian maximum *a posteriori* estimator to restore the clean coefficients by deriving an efficient closed-form shrinkage function. Experimental results are performed on a set of color images to evaluate the performance of the proposed denoising method. The results demonstrate that the proposed method outperforms some of the existing methods in terms of both subjective and objective criteria.**

Keywords— ***Image denoising, contourlet transform, multivariate Cauchy distribution, Bayesian MAP estimator.***

I. Introduction

Image denoising is becoming increasingly indispensable with the rapid growth of multimedia technology. The aim of a denoising task is to improve the perceptual quality of an image by removing the noise, while retaining the image features as much as possible. Recently, noise removal is mostly performed in the transform domain rather than in pixel domain [1]-[5]. The contourlet transform-based techniques, either by using a simple thresholding scheme or the Bayesian approach, have led to a remarkable success in image denoising as compared to the former transform domain-based methods [1].

In Bayesian approach, a shrinkage function is developed based on minimizing a Bayesian risk under the maximum *a posteriori* (MAP) criterion [1], [2]. The performance of the MAP estimator is highly determined by the accuracy of the prior for the contourlet coefficients. In [2] and [3], hidden Markov model has been used as a prior for the wavelet coefficients of images in an image denoising task. In [4], an image denoising method has been proposed based on Gaussian scale mixture model. In [5], a wavelet-based trivariate shrinkage filter for image denoising has been proposed by incorporating a spatial-based joint bilateral filter.

The contourlet subband coefficients of an image have shown to be non-Gaussian with heavy-tailed properties and can be described by the heavy-tailed distributions [6], [7]. In view of this, the marginal distribution of the contourlet coefficients of images have been previously modeled by the generalized Gaussian [8], normal inverse Gaussian (NIG) [9], [10] and alpha-stable distributions [6]. In [9], the NIG distribution has been used as a prior for contourlet coefficients in an image denoising task. In [11], a trivariate prior for the contourlet coefficients along with a non-local means filter has been proposed. In [12], a multivariate distribution has been proposed to model the inter-scale dependencies of the contourlet coefficients.

In case of denoising the color images, the most common approach is to employ the grayscale denoising method for each of the RGB color channels. However, it is known that there exists a considerable inter-channel dependency among the contourlet coefficients of the color images [13]. In this work, to formulate a closed-form MAP estimator for color image denoising, the contourlet subband coefficients of color images are modeled by the multivariate Cauchy probability density function (PDF) to capture both the heavy-tailed properties of the contourlet coefficients and their inter-channel dependencies.

II. Color Image Model

In image denoising task, it is known that the contourlet transform has led to a better performance than other transforms do [9], [10]. It is also known that the RGB channels of the color images are highly dependent [12]. The dependency among the contourlet domain coefficients of these channels plays an important role in modeling these coefficients. On the other hand, the contourlet coefficients of an image are non-Gaussian [6], [8]. In view of this, in this work, in order to model the contourlet coefficients of

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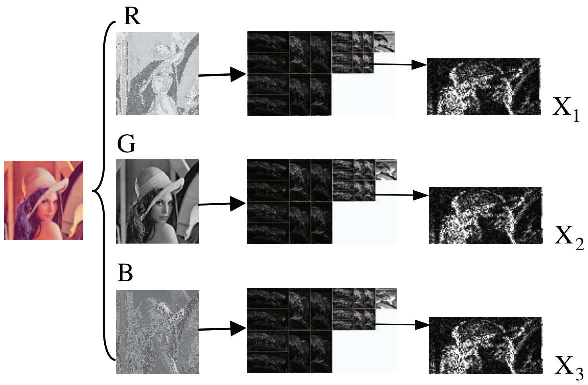

Fig. 1. Contourlet domain decomposition of the RGB channels for color images. Each subband coefficients X_1 are highly dependent to the same-oriented subband coefficients of two other channels, namely, X_2 and X_3.

color images, we propose the use of the multivariate Cauchy distribution to not only capture the heavy tails of their distribution but also to take into account their inter-channel dependencies. This model has been used before in an image watermarking task and shown to give promising results [13]. The PDF of the multivariate Cauchy distribution is given by

$$P_{\mathbf{x}}(\mathbf{x};\Sigma)=\frac{\Gamma(\frac{n+1}{2})}{\Gamma(\frac{1}{2})\pi^{0.5}\left|\Sigma\right|^{0.5}\left[1+\mathbf{x}^{T}\Sigma^{-1}\mathbf{x}\right]^{\frac{n+1}{2}}} \tag{1}$$

where n is the dimensionality of the distribution, i.e., the number channels, Σ is the covariance matrix that for the case of $n=3$ is given by

$$\Sigma=\begin{bmatrix}\sigma_1^2 & \rho_{12}\sigma_1\sigma_2 & \rho_{13}\sigma_1\sigma_3\\ \rho_{12}\sigma_1\sigma_2 & \sigma_2^2 & \rho_{23}\sigma_2\sigma_3\\ \rho_{13}\sigma_1\sigma_3 & \rho_{23}\sigma_2\sigma_3 & \sigma_3^2\end{bmatrix} \tag{2}$$

where $\rho_{ij};\ i,j=1,2,\dots,n$ denotes the correlation coefficient between each of the two color channels.

III. Bayesian MAP Estimator

An original color image $\mathbf{X}=[X_1,X_2,X_3]$ is corrupted by an additive noise $\mathbf{N}$ which is assumed to be i.i.d. Gaussian with zero mean and standard deviation σ_η. The corresponding noisy image $\mathbf{Y}=[Y_1,Y_2,Y_3]$ is given by $\mathbf{Y}=\mathbf{X}+\mathbf{N}$. The noisy image is decomposed into $k=1,\dots,K$ scales and $d=1,\dots,D$ directional subbands using the contourlet transform given by

$$y_{k,d}^{i}(m_1,m_2)=x_{k,d}^{i}(m_1,m_2)+\eta_{k,d}^{i}(m_1,m_2) \tag{3}$$

where $y_{k,d}^{i}(m_1,m_2)$, $x_{k,d}^{i}(m_1,m_2)$ and $\eta_{k,d}^{i}(m_1,m_2)$ denote the (m_1,m_2) th coefficient at scale k and direction d of the contourlet transform of $\mathbf{Y}$, $\mathbf{X}$ and $\mathbf{N}$, respectively. To have an estimate of the noise-free coefficients of the RGB channels, the Bayesian MAP estimator of $\mathbf{x}$, given noisy observation $\mathbf{y}$, is given by

$$\begin{aligned}\hat{\mathbf{x}}&=\arg\max_{\mathbf{x}} P_{\mathbf{x}|\mathbf{y}}(\mathbf{x}\,|\,\mathbf{y})\\&=\arg\max_{\mathbf{x}} P_{\mathbf{y}|\mathbf{x}}(\mathbf{y}\,|\,\mathbf{x})P_{\mathbf{x}}(\mathbf{x})\\&=\arg\max_{\mathbf{x}} P_{\eta}(\mathbf{y}-\mathbf{x})P_{\mathbf{x}}(\mathbf{x})\end{aligned} \tag{4}$$

where $P_{\mathbf{x}}(\mathbf{x})$ is the PDF of the contourlet coefficients of a noise-free image and $P_{\boldsymbol{\eta}}(\boldsymbol{\eta})$ is the PDF of the noise given by

$$P_{\boldsymbol{\eta}}(\boldsymbol{\eta})=\frac{1}{\left(\sqrt{2\pi}\sigma_\eta\right)^{n}}\exp\left(-\frac{\eta_1^2+\eta_2^2+\dots+\eta_n^2}{2\sigma_\eta^2}\right) \tag{5}$$

To estimate σ_η from the noisy contourlet coefficients, the estimator in [14] is modified in the finest decomposition scale as

$$\hat{\sigma}_\eta=\frac{1}{0.6745\,D}\sum_{d=1}^{D}\left(\mathrm{MAD}\{S_{1,d}\}\right) \tag{6}$$

where MAD is the *median absolute deviation* and $S_{1,d}$ denotes the d th directional subband coefficients in the finest scale. The parameters σ_i and ρ_{ij} of the noise-free coefficients can be estimated by using

$$\begin{aligned}\hat{\sigma}_i^2(m_1,m_2)&=\max\left(\sum_{S}\frac{y_i^2}{1^2}-\sigma_{\eta,i}^2\,,0\right)\\ \rho_{ij}&=\max\left(\min\left(\sum_{S}\frac{y_i y_j}{1^2\sigma_i\sigma_j},1\right),-1\right)\end{aligned} \tag{7}$$

where S is a square-shaped window of size 1×1 applied to estimate the variance of each signal coefficient in a given channel i. To obtain the MAP estimate, after inserting (5) into (4), the derivative of the logarithm of the argument in (4) is set to zero resulting in

$$\frac{\hat{x}_i - y_i}{\sigma_\eta^2} + \frac{\partial}{\partial y_i}\left(-\ln\left(P_{\mathbf{x}}(\mathbf{y})\right)\right) = 0 \tag{8}$$

An approximate solution of (8) is obtained as

$$\hat{x}_i = \mathrm{sign}(y_i).\left(\max\left(\left|y_i\right| - 4\sigma_\eta^2\left|B_i\right|\right), 0\right) \tag{9}$$

where B for each channel is given by

$$\begin{aligned} B_1 &= \frac{\sigma_1^2 y_1 + \rho_{12}\sigma_1\sigma_2 y_2 + \rho_{13}\sigma_1\sigma_3 y_3}{[1 + \mathbf{y}^T \Sigma^{-1} \mathbf{y}]} \\ B_2 &= \frac{\sigma_2^2 y_2 + \rho_{12}\sigma_1\sigma_2 y_1 + \rho_{23}\sigma_2\sigma_3 y_3}{[1 + \mathbf{y}^T \Sigma^{-1} \mathbf{y}]} \\ B_3 &= \frac{\sigma_3^2 y_3 + \rho_{13}\sigma_1\sigma_3 y_1 + \rho_{23}\sigma_2\sigma_3 y_2}{[1 + \mathbf{y}^T \Sigma^{-1} \mathbf{y}]} \end{aligned} \tag{10}$$

IV. Simulation Results

Experiments are performed on a set of standard color images of size 512×512 pixels, taken from http://sipi.usc.edu/database/, to evaluate the performance of the proposed denoising method and to compare the results with that provided by Bayes-shrink-luminance [1], BLS-GSM [4], TriSF-JBF [5], Surelet [15] and ProbShrinkMB [16]. The RGB channels of a noisy color image are first decomposed using the contourlet transform with 9-7 biorthogonal filters into two scales and eight directions in each scale. It is found through experiment that $1 = 5$ gives to an optimum denoising result. To overcome the shift-variant property of the contourlet transform, the cycle spinning mode is applied to all the detail coefficients. Table I gives the peak signal-to-noise-ratio (PSNR) values obtained using various denoising methods for three of the test images, namely, *Lena*, *Peppers* and *Baboon*. It is seen from this table that the proposed color image denoising method using the multivariate Cauchy prior yields generally better results than some of the existing methods except for TriSF-JBF method. Similar results have also been observed for other test images. The improved performance of TriSF-JBF is due using two times filtering first in luminance-chrominance color space, and then in RGB color space.

This improvement is however at the expense of over-smoothing the details. This over-smoothing diminishes the sharpness of the edges and results in a loss of some details. Fig. 2 illustrates the zoomed-in version of the noisy *Lena* image with $\sigma_\eta = 50$, as well as the denoised images obtained using various methods. It can be seen from this figure that the proposed color image denoising using the multivariate Cauchy prior provides denoised images with high quality while is capable of preserving more details as compared to other methods. Further, the over-smoothing caused by using the TriSF-JBF method is clearly perceptible from this figure, especially from the areas highlighted by the arrows. However, as compared to TriSF-JBF, the proposed color image denoising method can preserve more details.

TABLE I

PSNR values (in dB) obtained using various denoising methods for *Lena*, *Peppers* and *Baboon* images. (Best results in bold and second best in parentheses)

	Standard deviation			
Method	**10**	**15**	**20**	**25**
Lena				
Noisy image	28.13	24.62	22.13	20.16
Bayes-shrink-Luminance [1]	33.61	31.78	30.52	29.87
BLS-GSM [4]	34.45	32.90	31.78	30.89
Surelet [15]	34.64	33.02	31.90	31.04
ProbShrinkMB[16]	34.60	33.03	31.92	31.04
TriSF-JBF [5]	**35.14**	**33.80**	(32.90)	**32.17**
Proposed	(35.05)	(33.76)	**32.91**	(32.10)
Peppers				
Noisy image	28.13	24.61	22.11	20.17
Bayes-shrink-Luminance [1]	32.46	30.19	29.09	28.24
BLS-GSM [4]	33.26	31.89	30.92	30.13
Surelet [15]	33.35	31.79	30.72	29.89
ProbShrinkMB[16]	33.44	32.05	31.12	30.35
TriSF-JBF [5]	**33.86**	**32.63**	**31.84**	**31.17**
Proposed	(33.61)	(32.43)	(31.59)	(30.79)
Baboon				
Noisy image	28.13	24.61	22.11	20.17
Bayes-shrink-Luminance [1]	29.71	27.69	26.13	25.02
BLS-GSM [4]	30.13	27.66	26.08	24.95
Surelet [16]	30.49	28.15	26.64	25.55
ProbShrinkMB[16]	30.17	27.83	26.83	25.27
TriSF-JBF [5]	**30.86**	**28.65**	**27.20**	**26.16**
Proposed	(30.61)	(28.26)	(29.93)	(25.80)

V. Conclusion

In this work, a new contourlet domain color image denoising method has been proposed. A statistical model for the contourlet coefficients of the RGB color channels have been developed based on the multivariate Cauchy distribution to take into account both their heavy-tailed properties and inter-channel dependencies. The channels of noisy color image are decomposed into various scales and directional subbands via the contourlet transform. The MAP estimator utilizing the multivariate Cauchy prior

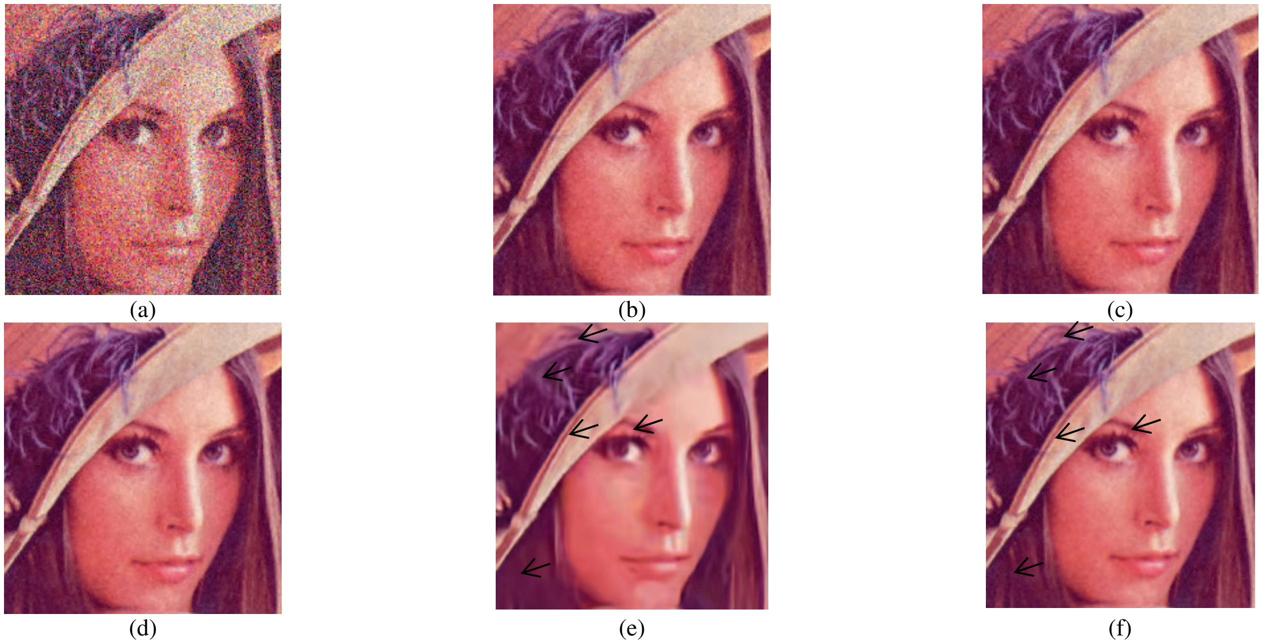

Fig. 2. Visual comparison of various denoising methods with $\sigma_\eta = 50$. (a) Noisy *Lena* image, PSNR = 14.18. (b) BLS-GSM, PSNR = 20.83. (c) Surelet, PSNR = 28.79. (d) ProbShrinkMB, PSNR = 28.93. (e) TriSF-JBF, PSNR = 29.72. (f) Proposed, PSNR = 29.61.

has been developed. To obtain an estimate of the noise-free coefficients in all the detail subbands of each channel, an efficient closed-form shrinkage function has been derived. Experiments have been carried out to evaluate the performance of the proposed denoising method for color images and to compare it with that of some of the existing methods. The results have shown that the proposed denoising method outperforms most of the existing methods in terms of the PSNR values and provides denoised images with high visual quality while preserving the details.

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