

# Optimum Multiplicative Watermark Detector in Contourlet Domain Using the Normal Inverse Gaussian Distribution

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**Abstract**— Digital watermarking has been widely used in the copyright protected images in multimedia. This paper addresses the blind watermark detection problem in contourlet domain. It is known that the contourlet coefficients of images have non-Gaussian property and can be well modelled by non-Gaussian distributions such as the normal inverse Gaussian (NIG). In view of this, we exploit this model to derive closed-form expressions for the test statistics and design an optimum blind watermark detector in the contourlet domain. Through conducting several experiments, the performance of the proposed detector is evaluated in terms of the probabilities of detection and false alarm and compared to that of the other existing detectors. It is shown that the proposed detector using the NIG distribution is superior to other detectors in terms of providing higher rate of detection. It is also shown that the proposed NIG-based detector is more robust than other detectors against attacks, such as JPEG compression and Gaussian noise.

**Keywords**— Contourlet transform, normal inverse Gaussian distribution, multiplicative watermark, blind watermark detection.

## I. INTRODUCTION

Digital watermarking is a process of embedding hidden secondary data into digital multimedia products for copyright notification and protection, content authentication, and secret communication [1]. There are many works on image watermarking in the transform domain by using the statistics of the subband coefficients of the wavelet transform [2], [3]. In recent years, the contourlet transform has received much attention as an alternative to other multi-scale and multi-resolution transforms in image watermarking [4], [5]. This is mostly due to its appealing characteristics in capturing smooth contours and geometric structures in images, and yielding sparser coefficients, i.e., it represents the edges by a few samples [6], [7]. In watermark verification process, a blind approach is more desirable since they need no access to the original image. In view of this and to design a blind watermark detector, watermark detection problem has been regarded as a statistical problem based on the statistics of the image coefficients [2], [4]. Researchers have studied the statistical properties of the contourlet subband coefficients and shown that the contourlet subband coefficients of images have

significantly non-Gaussian and heavy-tailed properties [6], [8]. The contourlet coefficients within a subband may be assumed to be independent and identically distributed. With this assumption, researchers have developed marginal models for the contourlet subband coefficients using the Cauchy distribution [4] and generalized Gaussian (GG) density [6]. In [4], we have developed the watermark detectors based on these distributions. It has been shown that the Cauchy based watermark detector is more robust than the GG-based detector and provides higher detection rates. In order to model the contourlet coefficients of images, we have proposed the normal inverse Gaussian (NIG) distribution [9]. It has been shown that this model provides a close fit to the distribution of the contourlet coefficients of images. In view of this in this work, we design a novel blind watermark detection scheme in the contourlet domain based on the NIG distribution. The performance of the proposed detector is evaluated by conducting several experiments and comparing it to that of the other existing methods. We observe that the watermark detection performance is improved as compared to the Cauchy and GG based detectors.

## II. WATERMARKING SCHEME

### A. Watermark embedding

There have been several works suggesting that the performance of the contourlet-domain algorithms is better than those based on other frequency-domain watermarking algorithms such as the wavelets [4] in terms of both invisibility of the watermark and resistance to attacks. This is mostly due to the spreading property of the contourlet transform in that if the watermark bits are inserted into specific subband, they will be spread out into all the subbands when the watermarked image is reconstructed. Suppose that an image is decomposed into  $J$  scale and  $D$  directional subbands by using the contourlet transform and subbands are denoted by  $S_{jd}$ , where  $j=1,\dots,J$  and  $d=1,\dots,D$ . In order to guarantee the imperceptibility of the watermark and taking into account the properties of the human visual system, the watermark bits are embedded into the directional subband coefficients of the finest scale with the highest energy as there are generally more edges in such subbands. In view of the fact

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that the multiplicative watermarks are image content dependent [10], and thus are more robust than additive watermarks, in our proposed watermarking scheme, we embed the watermark bits according multiplicative-spread spectrum rule as  $y_i = x_i(1 + \text{sign}(x_i)\xi)$ , where  $\{x_i\}_{i=1}^N$  and  $\{y_i\}_{i=1}^N$  are the original and watermarked coefficients, respectively,  $\xi \in \mathfrak{R}_+$  is a weighting factor and  $\{w_i\}_{i=1}^N$  is the bipolar watermark bits with elements  $\{-1, +1\}$  having equal probabilities. The weighting factor  $\xi$  can be calculated for an image using the watermark to signal ratio (WSD) given by  $\text{WSD} = 20 \log \frac{\xi}{\sigma_{x_i}}$ , where  $\sigma_{x_i}^2 = \frac{1}{N} \sum_i x_i^2$  [2], [4], [10]. In this way, the watermark can be adapted to the local properties of the original image. The watermarked contourlet coefficients are then inverse transformed to obtain the watermarked image.

### B. Watermark detection

The verification of the existence of the watermark, i.e., the watermark detection, is sometimes sufficient for the purpose of checking the authenticity of the copyright. In this work, we design a new watermark detector based on the NIG distribution as a prior for the contourlet coefficients of images. To this end, a Bayesian log-likelihood ratio test is employed for detecting the watermark. This method may be formulated as a binary hypothesis test where  $H_0: \xi = 0$  (not watermarked) and  $H_1: \xi \neq 0$  (watermarked) denote the null and alternative hypotheses, respectively. The decision rule is given by

$$\Lambda_{\text{det}} = \ln \left( \frac{f_y(Y | H_1)}{f_y(Y | H_0)} \right) > \tau \quad (1)$$

where  $\Lambda_{\text{det}}$  is the log-likelihood ratio and  $\tau$  is the threshold. The log-likelihood ratio can be formulated as

$$\begin{aligned} \mu_0 = E_w[\Lambda_{\text{det}} | H_0] &= -\frac{\ln(1+\xi) + \ln(1-\xi)}{2} \\ &+ \sum_{i=1}^N \left( \frac{\sqrt{(\delta^2 + y_i^2)} \left[ K_1 \left( \alpha \sqrt{\delta^2 + \left( \frac{y_i}{1 + \text{sign}(y_i)\xi} \right)^2} \right) K_1 \left( \alpha \sqrt{\delta^2 + \left( \frac{y_i}{1 - \text{sign}(y_i)\xi} \right)^2} \right) \right]^{1/2}}{\sqrt{\left( \delta^2 + \left( \frac{y_i}{1 + \text{sign}(y_i)\xi} \right)^2 \right) \left( \delta^2 + \left( \frac{y_i}{1 - \text{sign}(y_i)\xi} \right)^2 \right)} K_1 \left( \alpha \sqrt{\delta^2 + y_i^2} \right)} \right) \end{aligned} \quad (5)$$

and

$$\begin{aligned} \sigma_0^2 = \text{Var}[\Lambda_{\text{det}} | H_0] &= E_w[\Lambda_{\text{det}} - E[\Lambda_{\text{det}} | H_0]]^2 \\ &= \sum_{i=1}^N \frac{1}{4} \ln \left( \frac{\left( (1 + \text{sign}(y_i)\xi) \sqrt{\delta^2 + \left( \frac{y_i}{1 + \text{sign}(y_i)\xi} \right)^2} K_1 \left( \alpha \sqrt{\delta^2 + \left( \frac{y_i}{1 - \text{sign}(y_i)\xi} \right)^2} \right) \right)^2}{\left( (1 - \text{sign}(y_i)\xi) \sqrt{\delta^2 + \left( \frac{y_i}{1 - \text{sign}(y_i)\xi} \right)^2} K_1 \left( \alpha \sqrt{\delta^2 + \left( \frac{y_i}{1 + \text{sign}(y_i)\xi} \right)^2} \right) \right)^2} \right) \end{aligned} \quad (6)$$

$$\Lambda_{\text{det}} = \sum_{i=1}^N \ln \left( \frac{P\left(\frac{y_i}{1 + \text{sign}(y_i)\xi}\right)}{P(y_i)} \cdot \frac{1}{1 + \text{sign}(y_i)\xi} \right) \quad (2)$$

The data is modeled by an appropriate statistical distribution by assuming the independence of the observations. In this work, we exploit the zero-mean and symmetric NIG probability density function (PDF) given by

$$P(y) = \frac{\alpha \delta e^{\delta \alpha} K_1(\alpha \sqrt{\delta^2 + y^2})}{\pi \sqrt{\delta^2 + y^2}} \quad (3)$$

where  $\alpha$  and  $\gamma$  are the shape and scale parameters, respectively. The NIG parameters can be estimated using the moment generating function discussed in [9]. By inserting (3) into (2), the log-likelihood ratio becomes

$$\begin{aligned} \Lambda_{\text{det}} &= -\sum_{i=1}^N \ln(1 + \text{sign}(y_i)\xi) \\ &+ \sum_{i=1}^N \ln \frac{\sqrt{\delta^2 + y_i^2} K_1 \left( \alpha \sqrt{\delta^2 + \left( \frac{y_i}{1 + \text{sign}(y_i)\xi} \right)^2} \right)}{\sqrt{\delta^2 + \left( \frac{y_i}{1 + \text{sign}(y_i)\xi} \right)^2} K_1 \left( \alpha \sqrt{\delta^2 + y_i^2} \right)} \end{aligned} \quad (4)$$

Considering all terms of summation as independent, the log-likelihood ratio can be regarded as a superposition of  $N$  statistically independent random variables with finite mean and variance.



Fig. 1. (a) Original, (b) watermarked *Lena* image with PSNR= 65.23 dB, and (c) the difference between the original and watermarked image obtained using the proposed watermarking scheme.

Thus, according to the central limit theorem [12], the log-likelihood ratio under both the hypotheses can be approximated by the Gaussian distributions with means  $(\mu_0, \mu_1)$  and variances  $(\sigma_0^2, \sigma_1^2)$ . The mean and variance of the log-likelihood ratio under  $H_0$  where  $\forall i: y_i = x_i$ , is theoretically derived in (5) and (6). The mean and variance of the log-likelihood ratio under  $H_1$  can also be found in a similar manner. It can easily be shown that  $\mu_0 = -\mu_1$  and  $\sigma_0^2 = \sigma_1^2$ . Using these quantities, the probabilities of false alarm  $P_{fa}$  and detection  $P_{det}$  are related in a Neyman-Pearson sense [12] as

$$P_{det} = Q\left(Q^{-1}(P_{fa}) - \frac{2\mu_1}{\sigma_1}\right) \quad (7)$$

### III. SIMULATION RESULTS

Experiments are performed using a set of 100 grayscale images of size  $512 \times 512$  [13]. The images using the contourlet transform with 9-7 bi-orthogonal filters for both the multi-scale and multi-directional decomposition stages. To embed the watermark bits in the finest scale, the subband with the highest energy is selected. To evaluate the imperceptibility of the embedded watermark, the mean peak-signal-to-noise-ratio (PSNR) between the original and watermarked images are obtained by averaging over 10 runs with 50 different pseudo-random binary sequences as the watermark signal. The PSNR values for three of the test images, namely, *Barbara*, *Lena* and *Baboon*, are 59.41, 65.23, and 61.92 dB, respectively. Fig. 1 shows the original and watermarked *Lena* images, as well as the difference between the original and watermarked images. It is seen from this figure that the watermarking scheme is imperceptible. In order to evaluate the performance of the proposed watermark detector, we first examine the closeness of the theoretical and experimental ROC curves for the NIG, Cauchy and GG detectors. The experimental results are obtained from 1000 test runs for each image. Fig. 2 shows the experimental and theoretical ROC curves of different detectors for two of the test images, namely, *Barbara* and *Lena*.

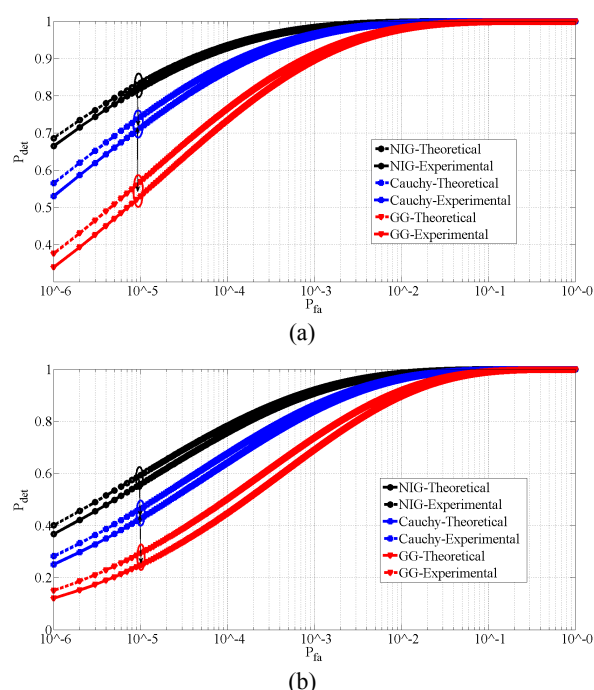


Fig. 2. The experimental and theoretical ROC curves using the NIG, Cauchy and GG based detectors for (a) *Barbara* (b) *Lena* images when WSR=-60

It is seen from this figure that the theoretical ROC curves are close to the empirical ones indicating the accuracy of the closed-form expressions in (5) and (6) for the mean and variance of the log-likelihood ratio. We then compare the detection performance of the proposed blind NIG-based watermark detector to that of the other detectors in terms of the probability of missing the watermark  $P_{miss} = 1 - P_{det}$ . Table I gives the probability of missing the watermark for the NIG, Cauchy and GG detectors for three of the test images, namely, *Barbara*, *Lena* and *Baboon*, when  $P_{fa} = 10^{-6}$ . It can be seen from this table that the proposed NIG-based detector performs better than other detectors by providing lower probabilities of miss detection.

Table I

Probability of missing the watermark  $P_{\text{miss}}$  for the NIG, Cauchy and GG-based detectors for various values of WSR.

	NIG	Cauchy	GG
WSR=-65			
Barbara	<b>0.4260</b>	0.6175	0.7769
Lena	<b>0.6843</b>	0.8126	0.9249
Baboon	<b>0.5671</b>	0.7497	0.8525
WSR=-60			
Barbara	<b>0.3021</b>	0.4264	0.6113
Lena	<b>0.5891</b>	0.7023	0.8919
Baboon	<b>0.4602</b>	0.5741	0.6955
WSR=-55			
Barbara	<b>0</b>	0.0831	0.1541
Lena	<b>0.0815</b>	0.1846	0.2248
Baboon	<b>0</b>	0.1172	0.1840

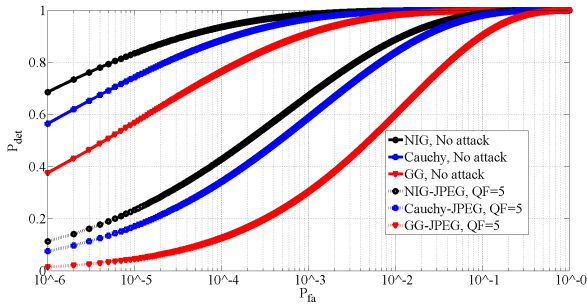


Fig. 3. ROC curves obtained using the NIG, Cauchy and GG-based detectors under JPEG compression attack with QF=5.

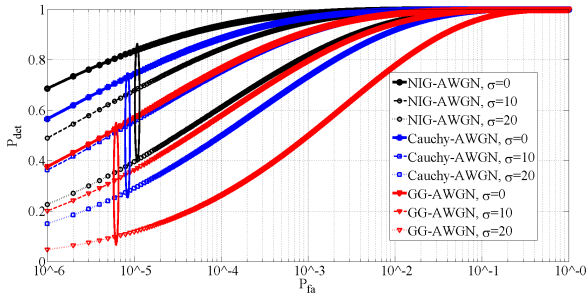


Fig. 4. ROC curves obtained using the NIG, Cauchy and GG-based detectors under additive Gaussian noise with various noise levels.

It should be noted that the same experimental setting is considered for different watermark detectors. We also examine the performance of the proposed detector against noise attacks. Fig. 3 show the ROC curves obtained using the proposed watermark detector when the watermarked *Barbara* image is JPEG-compressed with quality factor (QF) = 5. From this figure, it can be seen that the proposed NIG detector is more robust against JPEG compression attack than the Cauchy and GG detectors are. Fig. 4 shows the ROC curves obtained using the NIG, Cauchy and GG-based detectors when the Gaussian noise is added to the watermarked *Barbara* image. It is seen from this figure that for different levels of the Gaussian noise, the proposed NIG detector exhibits strong resistance against noise by providing higher detection rate for a given  $P_{fa}$  and hence, is more robust than other detectors under the Gaussian noise.

#### IV. CONCLUSION

In this work, we have proposed an optimum detector for the multiplicative watermarking scheme in contourlet domain by using the NIG distribution as a prior for the contourlet coefficients of images. The proposed detector has employed the Bayesian log-likelihood ratio criterion for the watermark detection. By the central limit theorem, the PDF of the log-likelihood ratio has been assumed to be Gaussian and closed-form expressions for the mean and variance of the log-likelihood ratio under null and alternative hypotheses have been derived. The performance of the proposed detector has been evaluated by conducting several experiments and compared to that of several existing detectors. It has been shown that the NIG-based watermark detector achieves increased detection performance as compared to other detectors. It has also been shown that the proposed watermarking scheme using the NIG-detector is more robust against different attacks than other detectors are.

#### REFERENCES

- [1] H. Sadreazami and M. Amini, "A robust spread spectrum based imagewatermarking in ridgelet domain," *International Journal on Electronics and Communications*, vol. 66, no. 5, pp. 364-371, 2012.
- [2] M. Amini, M. Omair Ahmad and M. N. S. Swamy, "A new blind wavelet domain watermark detector using hidden Markov model," *International Symposium on Circuits and Systems (ISCAS)*, pp. 2285-2288, 2014.
- [3] T. M. Ng and H. K. Garg, "Maximum-likelihood detection in DWT domain image watermarking using Laplacian modeling," *IEEE Signal Processing Letters*, vol. 12, no. 4, pp. 285-288, 2005.
- [4] H. Sadreazami, M. Omair Ahmad and M. N. S. Swamy, "A study of multiplicative watermark detection in the contourlet domain using alpha-stable distributions," *IEEE Transactions on Image Processing*, vol. 23, no.10, pp. 4348-4360, 2014.
- [5] A. Akhaee, S. M. Sahraei, and F. Marvasti, "Contourlet-based image watermarking using optimum detector in noisy environment," *IEEE Transactions on Image Processing*, vol. 19, no. 4, pp. 700-715, 2010.
- [6] M. N. Do, "Directional multiresolution image representations," Ph.D. diss., School Comput. Commun. Sci., Swiss Fed. Inst. Technol, 2001.
- [7] M. N. Do, and M. Vetterli, "The contourlet transform: an efficient directional multiresolution image representation," *IEEE Transactions on Image Processing*, vol. 14, no. 12, pp. 2091-2106, 2005.
- [8] H. Sadreazami, M. Omair Ahmad and M. N. S. Swamy, "Contourlet domain image modeling by using the alpha-stable family of distributions," *International Symposium on Circuits and Systems (ISCAS)*, pp. 1288-1291, 2014.
- [9] H. Sadreazami, M. Omair Ahmad and M. N. S. Swamy, "Contourlet domain image denoising using the normal inverse Gaussian distribution," *IEEE 27th Canadian Conference on Electrical and Computer Engineering (CCECE)*, pp. 1-4, 2014.
- [10] Q. Cheng and T. S. Huang, "Robust optimum detection of transform domain multiplicative watermarks," *IEEE Transactions on Signal Processing*, vol. 51, no. 4, pp. 906-924, 2003.
- [11] J. Hernandez, M. Amado, and F. Perez-Gonzalez, "DCT-domain watermarking techniques for still images: Detector performance analysis and a new structure," *IEEE Transactions on Image Processing*, vol. 9, no. 1, pp. 55-68, 2000.
- [12] A. Papoulis, *Probability, random variables, and stochastic processes*, New York: McGraw-Hill, 1991.
- [13] [Online]. Available: <http://decsai.ugr.es/cvg/dbimágenes/index.php>.