Despeckling of Synthetic Aperture Radar Images in the Contourlet Domain Using the Alpha-stable Distribution

H. Sadreazami, Student Member, IEEE, M. Omair Ahmad, Fellow, IEEE, and M. N. S. Swamy, Fellow, IEEE
Department of Electrical and Computer Engineering
Concordia University, Montreal, Quebec, Canada H3G 1M8
Email: {h_sadrea, omair, swamy}@ece.concordia.ca

***Abstract*—Speckle reduction has been a prerequisite for many SAR image processing tasks. This work presents a new approach for despeckling of SAR images in the contourlet domain using the alpha-stable distribution. It is shown that the alpha-stable distribution provides a good fit for the contourlet coefficients of an image, since it can capture the large peak and heavy tails of the distribution of the empirical data. This model is then exploited in a Bayesian maximum *a posteriori* estimator to restore the noise-free contourlet coefficients. The performance of the proposed despeckling method is evaluated using synthetically-speckled and real SAR images. Simulations are carried out using synthetically speckled images to investigate the performance of the proposed method, and compare it with that of some of the existing methods. The experimental results show that the proposed method can provide better preservation of the edges and can yield better visual quality as compared to some of the existing methods.**

***Index Terms*—SAR image denoising, contourlet transform, symmetric alpha-stable distribution, Bayesian MAP estimator.**

I. Introduction

Synthetic aperture radar (SAR) images are intrinsically affected by multiplicative speckle noise. There has been a growing effort in preprocessing of the SAR images due to their importance in many applications such as high-resolution remote sensing, surface surveillance and automatic target recognition [1], [2]. Therefore, a prerequisite for SAR imagery is to remove the multiplicative speckle noise. In recent years, multiscale transforms have been used with considerable success for recovering signal from noisy data [2]-[8]. It has been shown that the contourlet-domain denoising techniques have led to significant noise reduction in comparison to that provided by the earlier wavelet-based methods [9], [10]. This is due to the fact that the contourlet transform because of its flexible directional decomposability in each scale is more effective than wavelet is in representing smooth contour details in images [11].

It is known that the Bayesian estimators outperform classical linear processors and simple thresholding estimators in removing speckle noise from SAR images [3]. In view of this, the homomorphic filter-based methods wherein a suitable probability density function (PDF) is used as a prior model for describing the log-transformed wavelet coefficients have been proposed for multiplicative noise reduction [4]-[7]. In [5], a spatially-adaptive despeckling method has been proposed in wavelet domain. In [6], a dual-tree complex wavelet transform-based despeckling method has been used to denoise the SAR images. In [7], a Laplacian-Gaussian modeling have been used in wavelet domain for despeckling the SAR images.

The performance of the Bayesian estimator depends considerably on the correctness of the model assumed for the prior PDF of the image and noise. Since, the contourlet subband coefficients of an image have significant non-Gaussian and heavy-tailed statistics, a proper distribution to characterize the statistics of the contourlet coefficients would be a heavy-tailed PDF, such as the alpha-stable family of distributions [12]-[14]. In view of this, the objective of the present work is to introduce a new model for the contourlet coefficients of SAR images and to design a Bayesian MAP estimator using the symmetric alpha-stable distribution that exploits the statistics of the contourlet coefficients. Extensive simulations are conducted using synthetically-speckled and real SAR images, and the performance of the proposed method using the MAP estimator is compared to that of some of the existing techniques.

II. Modeling of the contourlet coefficients

In order to model the contourlet subband coefficients of an image, we propose the use of $S\alpha S$ PDF as a heavy-tailed non-Gaussian distribution to model the contourlet coefficients of the noise-free image. The $S\alpha S$ distribution with a random variable $X \sim S\alpha S(\alpha, \gamma)$ is described by its characteristic function [15]

$$\Phi_{\alpha,\gamma}(x) = \exp\left(-\gamma\left|x\right|^{\alpha}\right) \tag{1}$$

where α is a characteristic exponent,($0 < \alpha \leq 2$), and dispersion parameter $\gamma > 0$. In order to show the efficacy of the proposed prior, we investigate as to how accurately the alpha-stable distribution fits the distribution of the contourlet coefficients. For this purpose, we examine the histograms of the actual data as well as the $S\alpha S$, generalized Gaussian and

This work was supported in part by the Natural Sciences and Engineering Research Council (NSERC) of Canada and in part by the Regroupement Stratgique en Microlectronique du Qubec (ReSMiQ).

978-1-4799-8391-9/15/$31.00 ©2015 IEEE

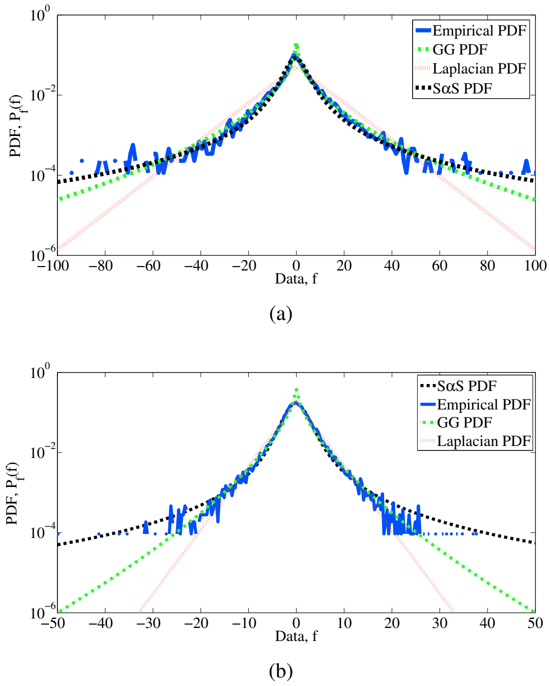

Fig. 1. Log-scale PDFs of empirical data as well as those of the $S\alpha S$, GG and Laplacian distributions for two directional subbands in the finest scale of the *Boat* image.

Laplacian PDFs for a set of test images. Fig. 1 illustrates the modeling performance of the contourlet coefficients for two directional subbands of one of the test images, the *Boat* image. It is evident from this figure that the $S\alpha S$ can more accurately fit the empirical data than the GG and Laplacian distributions can. Similar results are also obtained for other test images. The corresponding values of the Kolmogorov-Smirnov (KS) metric given by $max|\int P_f(f) - \hat{P}_f(f)df|$ in which, $P_f(f)$ denotes the PDF of the random variable and $\hat{P}_f(f)$ represents the PDF of the empirical data, are also obtained to examine the closeness of the empirical data to an assumed distribution. The value of KS metric is found to be 0.0492, 0.0943 and 0.1099 for the $S\alpha S$, GG and Laplacian distribution, respectively, indicating that the $S\alpha S$ distribution fits the empirical data more closely than the GG and Laplacian distributions do.

III. The Bayesian MAP estimator

Since the speckle noise model for SAR images is considered to be multiplicative, the observed output of the SAR imaginary system can be defined as

$$y_{i,j} = x_{i,j} n_{i,j} \tag{2}$$

where $y_{i,j}$ denotes the (i,j)th noisy pixel in a SAR image corresponding to the noise-free pixel $x_{i,j}$ and $n_{i,j}$ denotes the corrupting multiplicative speckle component. For an L-look SAR image, the PDF of the speckle noise has a gamma distribution given by

$$P_n(n) = \frac{L^L n^{L-1} e^{-Ln}}{\Gamma(L)} \tag{3}$$

where Γ denotes the gamma function and variance of noise equals to $\frac{1}{L}$ [5]. It should be noted that the lower the value of L, the higher is the level of the noise. With log-transformation, (2) becomes

$$Y_{i,j} = X_{i,j} + N_{i,j} \tag{4}$$

where $Y = ln(y)$, $X = ln(x)$ and $N = ln(n)$. The mean and variance of the logarithmically transformed gamma distribution are $\psi(0,L) - ln(L)$ and $\psi(1,L)$, respectively [5], where $\psi(i,L)$ is the ith polygamma function of L looks, and it is given by $\psi(i,z) = (\frac{\partial}{\partial z})^{i+1} ln\Gamma(z)$. Suppose that a noisy image is decomposed into $j = 1, ..., J$ scales and $d = 1, ..., D$ direction subbands by the contourlet transform. Then, we have $y_j^d(m,n) = x_j^d(m,n) + \eta_j^d(m,n)$, where $y_j^d(m,n)$, $x_j^d(m,n)$ and $\eta_j^d(m,n)$ denote the (m,n)th contourlet coefficient of the log-transformed noisy image at scale j with direction d, the corresponding log-transformed noise-free coefficient and the corresponding noise component after logarithmic transformation. Despeckling is based on estimating the noise-free coefficients x as a function of the noisy observations y. To this end, a Bayesian MAP estimator is developed through the modeling of the contourlet coefficients of a noisy image by the alpha-stable PDF. The Bayesian MAP estimator of x, given the noisy observation y, can be derived as

$$\begin{aligned} \hat{x}(y) &= argmax P_{x|y}(x|y) \\ &= argmax P_{y|x}(y|x) \\ &= argmax P_\eta(y-x)P_x(x) \end{aligned} \tag{5}$$

where $P_x(x)$ is the PDF of the contourlet coefficients of a noise-free image and $P_\eta(\eta)$ is the noise PDF. Although the speckle noise can be modeled by the gamma PDF, it becomes very close to the Gaussian distribution because of the logarithmic transformation. Therefore, in the proposed denoising method, the noise is assumed to be white Gaussian with a zero mean and a standard deviation of σ_η. The corresponding PDF is given by $P_\eta(\eta) = \frac{1}{\sqrt{2\pi\sigma_\eta^2}} \exp(-\frac{\eta^2}{2\sigma_\eta^2})$. If σ_η is unknown, it may be estimated by applying the robust median absolute deviation method [16] in the finest subband of the observed noisy coefficients. To obtain the MAP estimate, after inserting the noise PDF into (5), the derivative of the logarithm of the argument in (5) is set to zero resulting in

$$\frac{\hat{x} - y}{\sigma_\eta^2} + \frac{\partial}{\partial x}(-ln(P_x(x))) = 0 \tag{6}$$

Therefore, the Bayesian MAP estimator for non-Gaussian data is derived as

$$\hat{x}(y) = sign(y) max\left(0, |y| - \sigma_\eta^2 \left|\frac{\partial ln P_y(y)}{\partial y}\right|\right) \tag{7}$$

Since there is no closed-form PDF for the $S\alpha S$ distribution, we numerically compute the Bayesian MAP estimator output

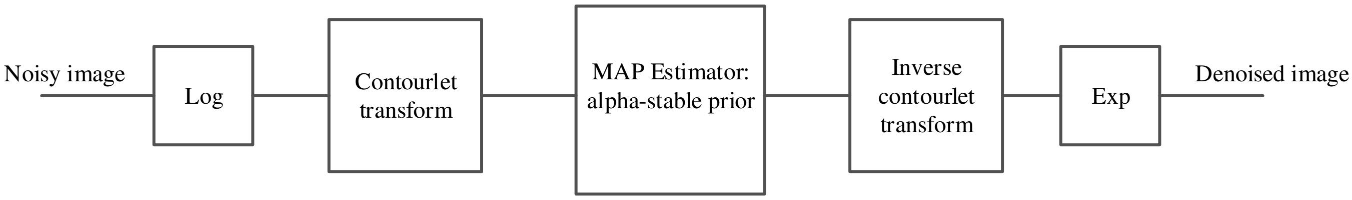

Fig. 2. Block diagram of the proposed algorithm for speckle reduction.

in (7). The proposed despeckling, whose block diagram is shown in Fig. 2, can be summarized as follows:

1) Perform the log-transformation of the observed SAR image.
2) Apply the contourlet transform on the log-transformed image and obtain the contourlet coefficients.
3) Estimate the parameters of the alpha-stable distribution γ and α from the noisy coefficients.
4) Estimate the noise-free coefficients using the Bayesian MAP estimator in (7).
5) Apply the inverse contourlet transform on the estimated noise-free coefficients.
6) Carry out the mean adjustment of the quantity obtained in Step 5 by subtracting it from the mean of log-transformed noise, i.e., $\psi(L) - ln(L)$.
7) Perform the exponential transformation of the values obtained in Step 6 to obtain the despeckled image.

IV. Simulation Results

The performance of the proposed despeckling method is evaluated by conducting experiments on synthetically-speckled or real SAR images. The obtained results are compared to those obtained by some of the existing speckle filtering methods in the wavelet and contourlet domain, namely, WIN-SAR [3], UWT [7], LAM [10], and NSCT [17]. It should be noted that the contourlet transform is a shift-variant transform. In order to overcome the possible pseudo-Gibbs phenomenon in the neighborhood of discontinuities, in the proposed despeckling method, the cycle spinning method [18] is performed on the observed noisy image. The noisy image is then decomposed, using the contourlet transform, into four scales with eight directions in each scale. In order to quantify the performance improvement, the peak signal-to-noise ratio (PSNR) is computed between the synthetically-speckled and denoised images.

Table I shows the PSNR values in decibels for the two synthetically-speckled images, namely *Boat* and *Lena*. It can be seen from this table that the proposed contourlet-based despeckling method provides PSNR values that are higher than those provided by the other methods for a given range of noise level. In case of the *Lena* image, the proposed despeckling method exhibits a clearly better performance than that of other methods do. Fig. 3 shows a real SAR image, *Ajkwa* image, obtained from the NASA/JPL website (http://airsar.jpl.nasa.gov) and its despeckled versions obtained by using the NSCT, UWT and LAM methods. The equivalent number of looks

TABLE I
PSNR VALUES OBTAINED USING DIFFERENT DESPECKLING METHODS FOR SYNTHETICALLY-SPECKLED *BOAT* AND *LENA* IMAGES. (BEST RESULTS IN BOLD, SECOND BEST IN PARENTHESIS)

	Number of looks (L)			
	3	4	5	6
	PSNR/BOAT			
Proposed	**23.05**	**23.89**	(24.32)	(24.56)
WIN-SAR [3]	22.14	23.13	23.92	24.42
NSCTS [17]	(22.93)	(23.79)	**24.35**	**24.68**
UWT [7]	21.83	22.49	23.07	23.88
LAM [10]	21.55	22.89	23.69	24.15
	PSNR/LENA			
Proposed	**28.95**	**30.08**	**30.72**	**31.46**
WIN-SAR [3]	25.09	26.93	29.05	30.62
NSCT [17]	(28.65)	(29.70)	(30.75)	(31.46)
UWT [7]	28.12	29.36	30.29	31.08
LAM [10]	25.78	26.73	27.29	27.90

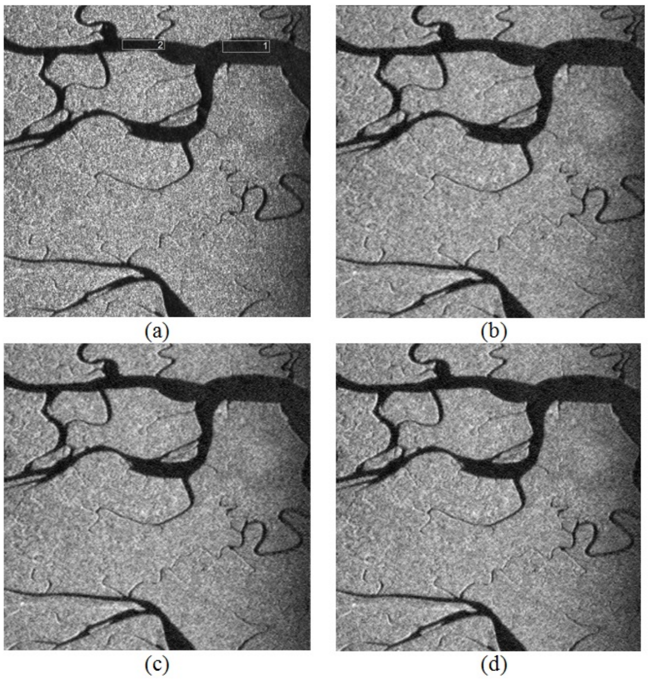

Fig. 3. (a)*Ajkwa* image. Obtained despeckled images by using various methods, namely, (b) WIN-SAR, (c) NCST, and (d) the proposed method.

TABLE II

ENL VALUES FOR TWO UNIFORM REGIONS IN THE *Ajkwa* IMAGE. (BEST RESULTS IN BOLD, SECOND BEST IN PARENTHESIS)

	Region 1	Region 2
Noisy	9.5	8.65
Proposed	(84.23)	**81.96**
WIN-SAR [3]	67.11	59.21
NSCT [17]	**85.12**	(81.60)
UWT [7]	79.27	49.04
LAM [10]	65.31	58.44

(ENL) value, defined by $ENL = \frac{\mu_x^2}{\sigma_x^2}$, is used to evaluate the performance of these methods for speckle noise reduction in which μ and σ are the mean and standard deviation within the selected homogenous region. For the ENL calculation, two uniform regions in the *Ajkwa* image are considered, shown in figure 3(a). Table II gives the ENL values obtained for two regions using various despeckling methods. From this table, it can be seen that the proposed method outperforms the other methods in reducing speckle noise in uniform regions and provides better visual quality by preserving the edges than those given by the other methods.

V. CONCLUSION

In this work, a new scheme for despeckling of SAR images in the contourlet domain has been proposed. The SAR images have been logarithmically transformed to convert the multiplicative speckle noise into an additive noise. It is then decomposed into various scales and directional subbands via the contourlet transform. The proposed method has been obtained by modeling the contourlet coefficients of the log-transformed SAR image using the symmetric alpha-stable distribution. It has been shown that this distribution can model the contourlet subband coefficients more accurately than formerly-used generalized Gaussian and Laplacian distributions can. The noise in all the detail subbands is removed by a Bayesian MAP estimator using the alpha-stable prior. Experiments have been carried out using the synthetically-speckled and real images to compare the performance of the proposed method with that of some of the existing methods. The results have shown that the proposed despeckling method outperforms other methods in term of the PSNR values and provides better visual quality despeckled images. Further, the proposed method has shown to provide a better speckle reduction in the homogeneous regions.

REFERENCES

[1] C. Oliver and S. Quegan, *Understanding Synthetic Aperture Radar Images,* Artech House, Boston, MA, 1998.

[2] Y. Bian, "Polarimetric SAR statistical analysis using alpha-stable distribution and its application in optimal despeckling," *International Journal of Remote Sensing,* vol. 34, no. 19, 6796-6836, 2013.

[3] A. Achim, E. E. Kuruoglu, and J. Zerubia, "SAR image filtering based on the heavy tailed Rayleigh model," *IEEE Transactions on Image Processing*, vol. 15, no. 9, pp. 2686-2693, 2006.

[4] M. I. H. Bhuiyan, M. O. Ahmad, and M. N. S. Swamy, "Spatially adaptive wavelet based method using the Cauchy prior for denoising the SAR images," *IEEE Transactions on Circuits Systems Video Technology,* vol. 17, no. 4, pp. 500-507, 2007.

[5] J. J. Ranjani and S. J. Thiruvengadam, "Dual-tree complex wavelet transform based SAR despeckling using interscale dependence," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 48, no. 6, pp. 386-397, 2010.

[6] F. Argenti, T. Bianchi, A. Lapini, and L. Alparone, "Fast MAP despeckling based on Laplacian-Gaussian modeling of wavelet coefficients," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 9, no. 1, pp. 13-17, 2012.

[7] F. Argenti, T. Bianchi, and L. Alparone. "Multiresolution MAP despeckling of SAR images based on locally adaptive generalized Gaussian pdf modeling," *IEEE Transactions on Image Processing,* vol. 15, no. 11, pp. 3385-3399, 2006.

[8] X. Tian, L. Jiao and X. Zhang, "Despeckling SAR images based on a new probabilistic model in nonsubsampled contourlet transform domain," *Signal, Image and Video Processing,* pp. 1-16, 2012.

[9] Q. Sun, L. Jiao, and B. Hou, "Synthetic aperture radar image despeckling via spatially adaptive shrinkage in the nonsubsampled contourlet transform domain," *Journal of Electronic Imaging,* vol. 17, no. 1, pp. 1-13, 2008.

[10] J. Sveinsson and J. Benedikisson , "Combined wavelet and contourlet denoising of SAR images," *IEEE International Geoscience and Remote Sensing Symposium (IGARSS),* pp. 1150-1153, 2008.

[11] M. N. Do, "Directional multiresolution image representations," Ph.D. dissertation, School Comput. Commun. Sci., Swiss Fed. Inst. Technol, 2001.

[12] H. Sadreazami, M. Omair Ahmad and M. N. S. Swamy, "Contourlet domain image modeling by using the alpha-stable family of distributions," *International Symposium on Circuits and Systems (ISCAS),* pp. 1288-1291, 2014.

[13] H. Sadreazami, M. Omair Ahmad and M. N. S. Swamy, "A study of multiplicative watermark detection in the contourlet domain using alpha-stable distributions," *IEEE Transactions on Image Processing,* vol. 23, no. 10, pp. 4348-4360, 2014.

[14] H. Sadreazami, M. Omair Ahmad and M. N. S. Swamy, "Contourlet domain image denoising using the alpha-stable distribution," *IEEE 57th International Midwest Symposium on Circuits and Systems (MWSCAS),* pp. 141-144, 2014.

[15] C. Nikias and M. Shao, *Signal processing with alpha-stable distributions and applications,* New York: Wiley, 1995.

[16] D. L. Donoho and I. M. Johnstone, "Ideal spatial adaptation by wavelet shrinkage," *Biometrika,* vol. 81, no. 3, pp. 425-455, 1994.

[17] F. Argenti, T. Bianchi, and L. Alparone, "SAR image despeckling in the undecimated contourlet domain: a comparison of LMMSE and MAP approaches," *International Geoscience and Remote Sensing Symposium (IGARSS)*, pp. 225-228, 2008.

[18] R. Eslami and H. Radha, "The contourlet transform for Image denoising using cycle spinning," In proc. *Asilomar Conference on Signals, Systems, and Computers,* vol. 2, pp. 1982-1986, 2003.