CONTOURLET DOMAIN IMAGE DENOISING USING THE ALPHA-STABLE DISTRIBUTION

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ABSTRACT

In this paper, a new contourlet-based method for denoising of images corrupted by additive white Gaussian noise is proposed. The alpha-stable distribution is used to model the contourlet coefficients of noise-free images. This model is then exploited to develop a Bayesian minimum mean absolute error estimator. A modified empirical characteristic function-based method is employed for estimating the parameters of the assumed alpha-stable prior. The performance of the proposed denoising method is evaluated by using standard noise-free images corrupted with simulated noise and compared with that of the other state-of-the-art methods. The results show that the proposed method provides values of the peak signal-to-noise ratio higher than that provided by some of the existing techniques along with superior visual quality images.

Index Terms— Contourlet transform, image denoising, alpha-stable distributions, MMAE estimator.

1. INTRODUCTION

Image denoising is a classical problem in signal processing. In general, a successful denoising algorithm should yield not only a reduction in noise, but also feature preservation. Image denoising in the wavelet domain has been addressed in recent years and shown to provide a satisfactory level of noise reduction [1]-[6]. It is known that a denoising method using Bayesian estimator provides a better performance in noise removal than conventional thresholding methods do [1], [2]. This improvement is due to fact that Bayesian estimator employs a prior for modeling the transformed coefficients. However, the performance of a thresholding method is highly dependent on the way the threshold is selected. In [4], the wavelet coefficients are denoised using a Bayesian minimum mean-squared error (MMSE) estimator based on modeling these coefficients with the generalized Gaussian (GG) distribution. In [5], the Jeffreys non-informative distribution is proposed to model the wavelet coefficients. Using this prior, a linear minimum mean-squared error is developed to denoise the coefficients. In [6], the Bessel-K distribution is used for developing a Bayesian MMSE estimator in order to reduce the noise from the wavelet coefficients. In [7], the symmetric normal Gaussian distribution is utilized to develop a MMSE estimator for denoising the dual-tree complex wavelet transform.

The contourlet-based image denoising methods have been shown to provide a significant improvement over the wavelet-based methods, and this is mostly due to the fact that the contourlet transform can capture more of the directional information and handle 2-D singularities [8], [9]. The objective of this paper is to design a Bayesian minimum mean absolute error (MMAE) estimator in contourlet domain for image denoising. The performance of the Bayesian estimator depends on the correctness of the contourlet coefficients prior. It is known that the contourlet subband coefficients of natural images have significant non-Gaussian and heavy-tailed properties that can be described by heavy-tailed distributions [8]-[10]. In this work, the contourlet coefficients of an image are assumed to be independent and identically distributed by the symmetric alpha-stable distribution (SαS). This model is suitable for describing signals that have highly non-Gaussian statistics and heavy tails [11], [12]. Thus, the MMAE estimator based on this distribution is developed and a numerical relation between the observed noisy and noise-free data is obtained resulting in a shrinkage function corresponding to the MMAE estimator. A modified empirical characteristic function-based method [13], is used for estimating the parameters of the SαS distribution from the noisy coefficients. The performance of the proposed denoising method is evaluated by using standard noise-free images corrupted with simulated noise and compared with that of the other state-of-the-art methods.

2. PROPOSED DENOISING METHOD

Let a noise-free image X be corrupted with an additive white Gaussian noise N with a zero-valued mean and known stan-

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dard deviation σ_η. If σ_η is unknown, it may be estimated by applying the robust median absolute deviation method [14] in the finest subband of the observed noisy coefficients. X and N are assumed to be independent. The corresponding noisy image, denoted by Y, is given by

$$Y = X + N \tag{1}$$

The contourlet transform is applied to the noisy image. Suppose a noisy image is decomposed into $j = 1, ..., J$ scales and $d = 1, ..., D$ direction subbands by the contourlet transform. Then, we have

$$y_j^d(m,n) = x_j^d(m,n) + \eta_j^d(m,n) \tag{2}$$

where $y_j^d(m,n)$ denotes the (m,n)th noisy coefficient of the image at scale j with direction d, corresponding to the noise-free coefficient $x_j^d(m,n)$, and $\eta_j^d(m,n)$ denotes the corrupting noise component. For notational simplicity, we drop the subscripts and indices as well, and use y, x and η throughout the rest of the paper. In this paper, we propose the use of the $S\alpha S$ distribution as a prior to model the contourlet coefficients corresponding to the noise-free image. A random variable $X \sim S\alpha S(\alpha,\gamma)$ is described by its density function [12], [13]

$$f_{\alpha,\gamma}= \begin{cases} \frac{\alpha}{\pi\gamma^{1/\alpha}}\sum_{k=1}^{\infty}\frac{(-1)^{k-1}}{(k-1)!}\Gamma(\alpha k)\sin(\frac{k\alpha\pi}{2})(\frac{|x|}{\gamma^{1/\alpha}})^{-\alpha k-1} & 0<\alpha<1 \\ \frac{\gamma}{\pi(x^2+\gamma^2)} & \alpha=1 \\ \frac{1}{\pi\gamma^{1/\alpha}}\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k)!}\Gamma\big(\frac{2k+1}{\alpha}\big)\big(\frac{x}{\gamma^{1/\alpha}}\big)^{2k} & 1<\alpha<2 \\ \frac{1}{2\sqrt{\pi\gamma}}\exp(-\frac{x^2}{4\gamma}) & \alpha=2 \end{cases} \tag{3}$$

where α is a characteristic exponent, $(0 < \alpha \leq 2)$, and dispersion parameter $\gamma > 0$. The characteristic exponent α is the most important parameter, which determines the shape of the distribution. The smaller the value of α, the heavier the tail of the distribution. This implies that random variables following the $S\alpha S$ distribution with small characteristic exponents are highly impulsive. In order to show the efficacy of the proposed prior, we investigate as to how accurately the alpha-stable distribution fits the distribution of the contourlet coefficients. For this purpose, we examine the cumulative density function (CDF) of the actual data as well as those of the SαS and generalized Gaussian PDFs for a set of test images. Fig. 1 illustrates the modeling performance of the contourlet coefficients for one of the images, the *Barbara* image. It is seen from this figure that the SαS provides a more accurate fit to the empirical data than the GG distribution does. Similar results are also obtained for other test images. The corresponding values of the Kolmogorov-Smirnov (KS) metric given by $max|\int P_f(f) - \hat{P}_f(f)df|$, in which $P_f(f)$ denotes the PDF of the random variable and $\hat{P}_f(f)$ represents the PDF of the empirical data, are also obtained to

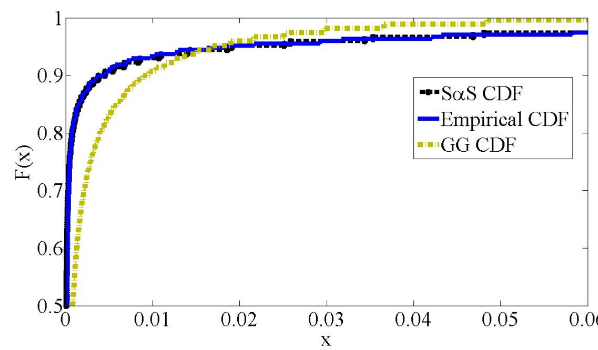

Fig. 1: CDFs of empirical data as well as those of the $S\alpha S$ and GG distributions for the *Barbara* image.

objectively examine the closeness of the empirical data to an assumed distribution. The value of KS metric is found to be 0.0867 and 0.1358 for the SαS and GG distributions, respectively, indicating that the SαS distribution fits the empirical data more closely than the GG distribution does. In order to employ the SαS prior in the proposed denoising method, first we need to estimate the parameters α and γ from the noisy coefficients. To this end, a modified empirical characteristic function-based (MECF) method is employed in which the associated characteristic function of the observed noisy coefficients is expressed as a product of the characteristic functions of the noise-free coefficients and the noise as

$$\phi_y(\omega) = \phi_x(\omega)\phi_\eta(\omega) \tag{4}$$

where

$$\begin{aligned} \phi_x(\omega) &= \exp\left(-\gamma|\omega|^\alpha\right) \\ \phi_\eta(\omega) &= \exp\left(-\frac{\sigma_\eta^2}{2}|\omega|^2\right) \end{aligned} \tag{5}$$

The MECF method is based on minimizing the sum of squared errors, which is defined in the least-square sense. In order to decrease the computational complexity of minimization algorithm, we use natural logarithms of characteristic functions. The cited error is then defined as the difference between the empirical characteristic function, $\hat{\phi}_{ye}(\omega) = \frac{1}{N}\sum_{n=1}^{N}\exp(jy_n\omega)$ obtained from the observed data $y_j^d(m,n)$ of length N, and the characteristic function expression given by (4) at some specific frequencies ω_m. Then, we have

$$\{\alpha,\gamma\} = min\left\{\sum_{m=1}^{M}\left\|ln\hat{\phi}_{ye}(\omega_m) - ln\phi_y(\omega_m)\right\|^2\right\} \tag{6}$$

where M is the total number of frequencies considered in the least squares method and $\|.\|$ indicates the Euclidean norm.

Using the Taylor series expansion, (6) can be rewritten as

$$\{\alpha, \gamma\} = min\left\{\sum_{m=1}^{M}\left\|\sum_{i=0}^{\infty}\hat{c}_i(j\omega_m)^i - \sum_{i=0}^{\infty}c_i(\omega_m)^i\right\|^2\right\} \tag{7}$$

where

$$\hat{\phi}_{ye}(\omega) = \sum_{k=0}^{\infty}\frac{a_k(j\omega)^k}{k!}, \quad a_k \cong \frac{1}{N}\sum_{n=1}^{N}x_n^k \tag{8}$$

To approximate (6), we consider the first few terms of the Taylor series. Since the coefficients in the approximation subband carry most of the information about the signal to be recovered, we leave them unchanged. In order to estimate the noise-free coefficients in the contourlet domain, a Bayesian MMAE estimator is developed through the modeling of the contourlet coefficients of the noisy image based on the proposed SαS prior, followed by minimizing the mean absolute error between the observed data and the estimated one. The Bayesian MMAE estimator of x, given the noisy observation y can be derived as

$$\hat{x}(y) = \int xP_{x|y}(x|y)dx \tag{9}$$

According to the Bayesian rule, $P_{x|y}(x|y)$ can be written as

$$P_{x|y}(x|y) = \frac{P_{y|x}(y|x)P_x(x)}{\int P_{y|x}(y|x)P_x(x)dx} \tag{10}$$

where $P_x(x)$ is the PDF of the contourlet coefficients of a noise-free image. Substituting (10) into (9), we obtain

$$\begin{aligned}\hat{x}(y) &= \frac{\int xP_{y|x}(y|x)P_x(x)dx}{\int P_{y|x}(y|x)P_x(x)dx} \\ &= \frac{\int xP_\eta(\eta)P_x(x)dx}{\int P_\eta(\eta)P_x(x)dx}\end{aligned} \tag{11}$$

where $P_\eta(\eta)$ is the PDF of the noise. In order to estimate the noise-free coefficients, we numerically compute the Bayesian estimator in (11). The proposed method can be summarized as follows.

1. Apply the contourlet transform on the noisy image and obtain the contourlet coefficients.
2. Estimate the parameters of the $S\alpha S$ distribution, γ and α, from the noisy coefficients by using (7).
3. Estimate the noise-free coefficients of all detail subbands using the Bayesian MMAE estimator in (11).
4. Apply the inverse contourlet transform on the estimated noise-free coefficients to obtain the denoised image.

Table 1: PSNR VALUES OBTAINED USING DIFFERENT DENOISING METHODS FOR TWO OF THE TEST IMAGES, THE *BARBARA* AND *PEPPERS* IMAGES

	Standard deviation			
Method	10	15	20	25
	Barbara			
Noisy image	28.13	24.61	22.11	20.17
Proposed	**31.57**	29.81	**28.51**	**27.34**
Visu-shrink(soft)	27.34	24.44	22.19	20.06
Visu-shrink(hard)	28.78	26.85	25.46	24.46
Adaptive-shrink	-	**29.96**	28.36	27.23
SURE-shrink	28.20	24.64	22.13	20.19
Bayes-shrink	30.28	27.51	26.09	25.56
NIG-CT	30.77	-	27.05	-
	Peppers			
Noisy image	28.13	24.61	22.11	20.17
Proposed	**32.37**	**31.05**	**29.43**	**27.29**
Visu-shrink(soft)	29.70	27.88	25.31	23.20
Visu-shrink(hard)	29.34	28.12	27.26	26.50
Adaptive-shrink	31.93	30.01	28.37	27.23
SURE-shrink	28.20	24.65	22.13	20.18
Bayes-shrink	30.97	29.63	28.94	26.85
NIG-CT	29.72	-	27.32	-

3. SIMULATION RESULTS

The performance of the proposed denoising method is evaluated by conducting experiments on some standard images and comparing to that of some of the existing methods, namely, adaptive-shrink [1], Bayes-shrink [2], Visu-shrink [15], SURE-shrink [16] and NIG-CT [17]. The experiments are performed on the images corrupted with various levels of Gaussian noise, σ varying from 10 to 25. The noisy image is transformed by the contourlet transform with three levels of decomposition and 8, 8 and 4 directions from the finer to coarser scales, respectively. Since the contourlet transform is a shift-variant transform, in the proposed contourlet domain denoising, to overcome the possible Pseudo-Gibbs phenomena in the neighborhood of discontinuities, the cycle spinning method [18] is performed on the observed data. The peak signal-to-noise ratio (PSNR) is employed as an objective performance criterion. Table I gives the PSNR values obtained using the proposed denoising method and some of the other existing methods for two of the test images, *Barbara* and *Peppers*. It is seen from this table that the proposed method provides a generally higher PSNR value, for a given range of noise standard deviation. Similar results are also observed for other test images. Fig. 2 shows the denoised *Barbara* image obtained using various denoising methods. It is seen from this figure that the proposed denoising method provides a visual quality superior to that yielded by some of the other existing methods.

Fig. 2: Visual comparison of various denoising methods with . (a) Original *Barbara* image. (b) Noisy image, PSNR= 22.11. (c) Visu-shrink(hard) PSNR=25.46. (d) Bayes-shrink, PSNR=26.09. (e) adaptive-shrink, PSNR= 28.36. (f) Proposed, PSNR= 28.51.

4. CONCLUSION

In this paper, we have proposed a new image denoising method in the contourlet domain. The proposed method has been carried out by modeling the contourlet transform using the symmetric alpha-stable distribution. It has been shown that this distribution provides a more accurate modeling of the contourlet coefficients than the formerly-used distributions in this domain. A modified empirical characteristic function-based method has been used for estimating the parameters of the assumed distribution. A Bayesian MMAE estimator has been developed using the proposed statistical prior in order to denoise the contourlet coefficients. Experiments have been carried out to compare the performance of the proposed method with that of the some of the existing methods. The simulation results have shown that the proposed denoising method outperforms some of the existing methods in terms of PSNR values and provides the denoised images with a superior visual quality.

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