KEY PARAMETERS INFLUENCE ON WAVELET BASED OFDM SYSTEMS

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ABSTRACT

In this paper, we investigate the use of orthogonal wavelet basis functions as an alternative to multicarrier systems based on fast Fourier transform. The influence of the key parameters such as the wavelet decomposition level, the wavelet basis family and the filter order on the receiver detection performance is studied in comparison to the conventional orthogonal frequency division multiplexing.

Numerical results conducted over both AWGN and multipath fading channels show that wavelet based OFDM outperforms conventional OFDM in multipath channels in terms of bit error rate. This improvement is examined in the face of different wavelet filter orders and decomposition levels.

Index Terms— OFDM, Wavelet-based OFDM, discrete wavelet transform parameters, multipath fading channel.

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is a mature technology for spectrally efficient data transmission which has attracted great research interests in the past twenty years [1, 2].

In classical OFDM systems, fast Fourier transform (FFT) is used for data modulation. This scheme divides the high data rate bit stream into lower data rate sub streams, so that symbol duration in each sub stream is much greater than the channel delay spread.

However, the major drawback of OFDM systems is the power leakage in the frequency domain due to the side-lobes of the shaping filter. This power leakage leads to harmful interference among subcarriers, especially in the presence of frequency offset.

This observation motivated the use of other orthogonal basis functions with more compact spectrum. Wavelet based OFDM, referred to as WOFDM, is a good alternative for Fourier based OFDM systems due to the high flexibility in the spectrum of the wavelet basis functions [3-6].

Unlike OFDM, WOFDM systems do not require the insertion of a cyclic prefix (CP) at the transmitter, though it could be used to increase the performance, so higher spectral efficiency is achieved. In fact, the amount of the improvement provided by WOFDM highly depends on key parameters of the wavelet transform. More precisely, the shape of the spectrum depends on parameters such as the wavelet family, the wavelet filter order and the wavelet decomposition level [7].

In [8], a closed-form expression for convolutions counterpart in the wavelet domain has been derived along with a comparison between WOFDM and OFDM over ultra wideband channels. However, there is no work on the simultaneous effect of all wavelet transform parameters affecting WOFDM detection performance.

In this paper, we first compare the performance of WOFDM with conventional OFDM over both AWGN and multipath channels. Then, we consider different orthogonal wavelet families in order to find out the best wavelet family in terms of system bit error rate (BER). We also investigate the effect of the wavelet decomposition level and the filter order on the WOFDM detection performance.

The rest of the paper is organized as follows. In Section II we provide a brief survey on the theory of discrete wavelet transform. In Section III, we describe the system models for both conventional OFDM and WOFDM. Simulation results and discussion are presented in Section IV.

II. OVERVIEW OF DISCRETE WAVELET TRANSFORM

The discrete wavelet transform (DWT) of a sequence $s[n]$ of length N may be calculated as a series of convolutions and decimations [9, 10]. At each decomposition level j, an input sequence $c^{j-1}[n]$ is fed into low pass and high pass filters with coefficients given by $G[n]$ and $H[n]$, respectively. The DWT starts with setting $c^0[n] = s[n]$.

The output from the high pass filter $H[n]$ is referred to as detail coefficients which are denoted by $d^j[n]$ and the output from the low pass filter $G[n]$ represents the coarse or approximate coefficients, which are denoted by $c^j[n]$. We have:

$$c^j[n] = \sum_k c^{j-1}[k]\, G[2n-k] \tag{1}$$

and

$$d^j[n] = \sum_k d^{j-1}[k]\, H[2n-k] \tag{2}$$

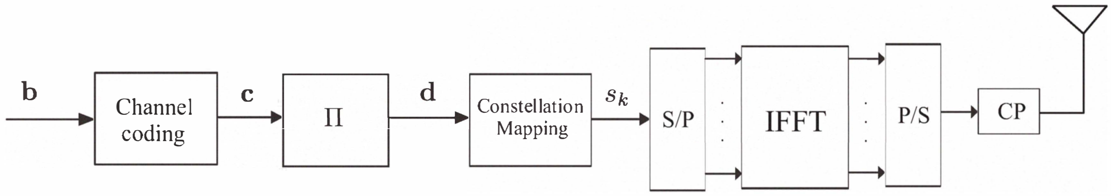

Fig. 1. Block diagram of the conventional OFDM transmission system.

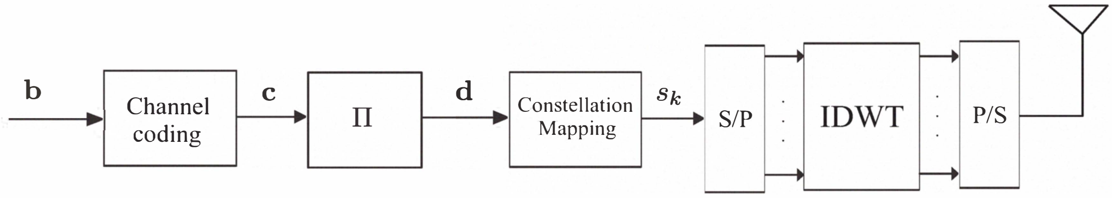

Fig. 2. Block diagram of the WOFDM transmission system.

where $k = 0, ..., N/2 - 1$ and $j = 0, ..., J - 1$, with J being the maximum level of wavelet decomposition. The original signal is recursively reconstructed by inverse discrete wavelet transform (IDWT) as follows:

$$c^{j-1}[n] = \sum_k c^j[k]\,G[2n-k] + d^j[k]\,H[2n-k] \qquad (3)$$

III. FOURIER-BASED OFDM VERSUS WAVELET-BASED OFDM

We consider a coded OFDM system with M subcarriers through a frequency selective multipath fading channel impulse response, described in discrete time baseband equivalent form by $\{h_l\}, l = 0, ..., L-1$, where L is the maximum channel delay spread.

As depicted in Fig. 1, the binary data sequence is encoded by a forward error correction code, before being interleaved by a pseudo-random interleaver. The interleaved bits are then mapped to complex QPSK symbols s_k. In order to avoid inter symbol interference, due to the multipath channel, CP is added to the generated signal.

At the receiver, after removing the CP and performing FFT, the the n-th received OFDM symbol vector can be written as [2]:

$$\mathbf{y} = \text{diag}\{\mathbf{H}\}\,\mathbf{s} + \mathbf{z}, \qquad (4)$$

where $\mathbf{y} = [y_0, ..., y_{M-1}]^T$ and $\mathbf{s} = [s_0, ..., s_{M-1}]^T$ respectively denote the received and transmitted symbol vectors, $\mathbf{z} = [z_0, ..., z_{M-1}]^T$ is the noise vector and diag$\{\mathbf{H}\}$ is the $(M \times M)$ diagonal channel matrix with diagonal elements given by the vector $\mathbf{H} = [H_0, ..., H_{M-1}]^T$ where $H_k = \sum_l^{L-1} h_l\, e^{-j2\pi kl/M}$.

The use of wavelet transform in multicarrier systems is an alternative way to generate subcarriers and divide the available bandwidth into smaller subbands.

It is well known that conventional OFDM with M subcarriers divides the total bandwidth into M subbands of equal bandwidths. However, in WOFDM, the number of subbands and their bandwidth can be dynamically tuned according to the wavelet transform decomposition level. This features makes WOFDM particularly suitable for *cognitive radio* systems [11] where spectral flexibility is of great importance.

As shown in Fig. 2, the transmitter architecture of the WOFDM system is similar to that of the conventional OFDM, except that there is no CP insertion and the IFFT is replaced by the IDWT. The received vector $\mathbf{y}$ can thus be written as:

$$\mathbf{y} = \mathbf{C}\,\mathbf{W}_i^H\,\mathbf{s} + \mathbf{z}, \qquad (5)$$

where $\mathbf{C}$ is the linear convolution matrix formed from the sequence $\{h_l\}$ and $\mathbf{W}_i^H$ is the inverse wavelet transform matrix where i represents a given set of wavelet transform parameters.

More precisely, parameters such as the type of the wavelet family, the filter order and the decomposition level determine the entries of the i-th matrix $\mathbf{W}_i^H$. It is well known that the matrix $\mathbf{C}$ can be diagonalized by the Fourier basis, i.e., $\mathbf{C} = \mathbf{F}^H\,\text{diag}\{\mathbf{H}\}\,\mathbf{F}$. In this way, by using a zero-forcing equalizer for instance, the estimated symbol vector $\hat{\mathbf{s}}$ can be obtained as:

$$\hat{\mathbf{s}} = \mathbf{W}_i\,\mathbf{F}^H\,\text{diag}\{\mathbf{H}\}^{-1}\mathbf{F}\,\mathbf{y}, \qquad (6)$$

where $\mathbf{W}_i$ and $\mathbf{F}$ stand respectively for DWT and FFT transform matrix and $(.)^H$ denotes conjugate-transpose.

IV. SIMULATION RESULTS AND DISCUSSIONS

In this section, we provide numerical results to evaluate the performance provided by WOFDM in comparison to conventional OFDM. Throughout the simulations, one (W)OFDM symbol is composed of $M = 128$ subcarriers. The interleaver is pseudo-random and operates over the entire frame of size 64 (W)OFDM symbols. Data symbols belong to the QPSK constellation.

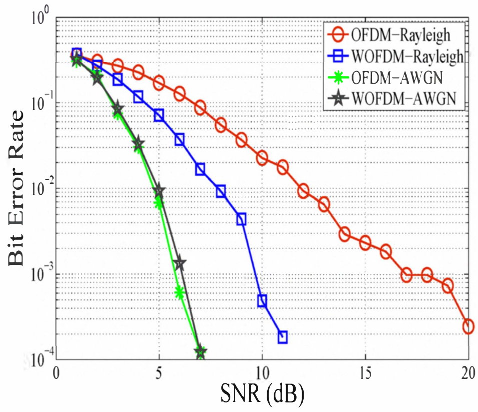

Fig. 3. BER performance improvement over Rayleigh fading channels by using WOFDM, *Symlet family*, $J = 3$.

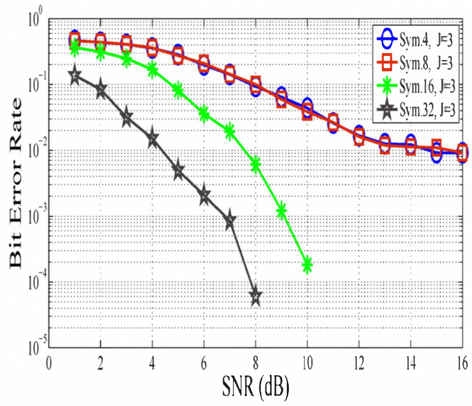

Fig. 5. Improvement of the WOFDM BER by increasing the wavelet filter order, *Symlet* wavelets, $J = 3$, Rayleigh fading channel.

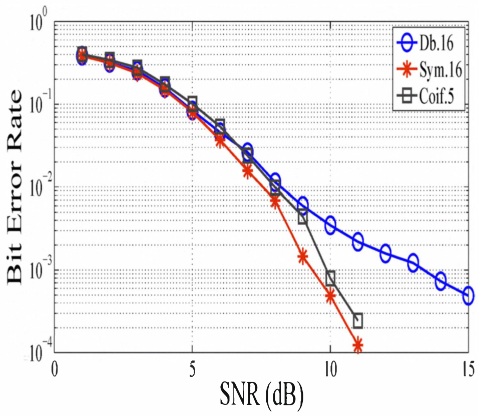

Fig. 4. BER performance of WOFDM with different wavelet families, $J = 3$, Rayleigh fading channel.

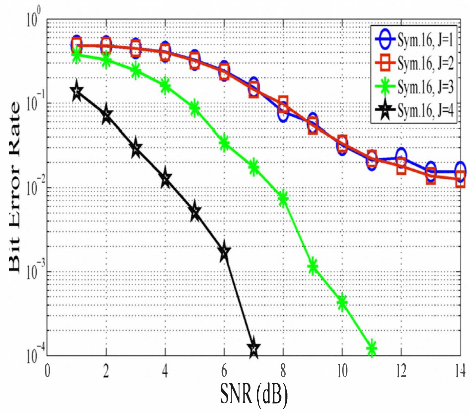

Fig. 6. Improvement of the WOFDM BER by increasing the wavelet transform decomposition level j, *Symlet* wavelets, $J = 3$, Rayleigh fading channel.

The performance evaluation is performed over two channels: i) the AWGN channel, and ii) the uncorrelated Rayleigh fading channel. Assumptions are made that the simulation is conducted without considering the channel estimation.

First, the BER of conventional OFDM and WOFDM are compared. It can be observed from Fig. 3 that WOFDM largely outperforms OFDM over Rayleigh channels. Moreover, due to the orthogonality of wavelet and Fourier transforms, both schemes have similar performance over AWGN channels. This main observation motivates us to find the best wavelet transform parameters leading to more performance improvement.

Figure 4 depicts the BER of the WOFDM systems obtained with different wavelet families. It can be seen that in the considered simulation, the *Symlet* family leeds to a lower BER performance due to its more symmetric waveform. Hence, *Symlet* wavelets are considered in the following experiences.

Figure 5 shows the impact of increasing the wavelet filter order in a given wavelet family. For instance, it is observed that increasing the filter order from 4 to 32 in *Symlet* wavelets, leads to a significant reduction in the SNR level. This is probably because of increasing orthogonality among wavelet coefficients and the gain is almost the same for all wavelet filters. Similar plots are shown in Fig. 6 contrasting the BER of WOFDM with respects to the decomposition level J for *Symlet* wavelets. The conclusion is quite similar to Fig. 5. However, it is worth to understand which of the above two parameters has the more important impact on the WOFDM performance. To this end, Fig. 7 shows the behavior of the WOFDM system with respect to both the decomposition level and the filter order. It can be seen that WOFDM benefits more from the increase in the decomposition level than the increase in the filter order.

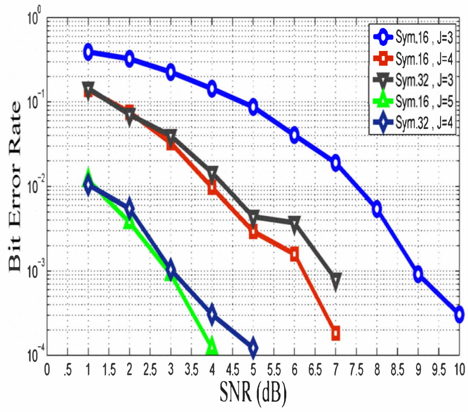

Fig. 7. Behavior of the WOFDM system with respect to the decomposition level and the wavelet filter order, *Symlet* wavelets, Rayleigh fading channel.

V. CONCLUSION

The spectral flexibility and the higher data rates of WOFDM systems make them a good alternative to conventional OFDM systems. In addition, WOFDM outperforms conventional OFDM in terms of BER over Rayleigh fading channels.

Our results showed the robustness of *Symlet* wavelets over multipath fading channels. The amount of WOFDM bit error rate improvement was shown to be more important when the filter order and/or the decomposition level are increased. Thanks to fast wavelet transform algorithms, the reported performance improvements for WOFDM was obtained without requiring additional complexity in the receiver.

In future work, we plan to explore about implementation and computational complexity using different wavelet filters as they all have their own advantages in the face of certain application situations.

VI. REFERENCES

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