To Study an Accurate Method for Analysis of Substrate Integrated Waveguide

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Abstract **— In this paper, an accurate analysis method of substrate integrated waveguide based on mode matching technique is presented. The structure is approximated by consecutive rectangular waveguides. A unit cell structure is analyzed by considering field distribution as the summation of TE_{n0} modes in rectangular coordinate system. Due to the periodicity of the structure, the propagation characteristics of substrate integrated waveguide modes can be deduced from investigating of the unit cell and employing Floquet theory. Finally, through comparison between our results with other published results, accuracy of the proposed method is verified.**

Index Terms **— Substrate integrated waveguide, Mode matching method, Floquet's theorem.**

I. Introduction

In multi-layer microwave integrated circuits such as low temperature co-fired ceramics or multi-layer printed circuit boards substrate integrated waveguides (SIW) are fabricated as an alternative structure to conventional transmission lines [1, 2]. Substrate integrated waveguides are synthetic rectangular waveguides formed by top and bottom metal layers which embed a dielectric slab and two sidewalls of metallic vias, largely preserving advantages of metallic waveguides (e.g., high quality-factor, high power handling capability, etc.)

Also, substrate integrated waveguide is easy to integrate with planar circuit which makes it a good choice for microwave and millimeter wave components such as filters, resonators, mixers and antennas. Various numerical techniques such as mode matching method [3], finite-difference frequency-domain [4], method of moment [5] and method of lines [6] have been applied to analyze the substrate integrated waveguide structures.

In [3] cylindrical via holes are approximated to rectangles and the propagation properties of the structure are investigated. Differentiation between dispersion diagram and empirical formula in [7] was due to the approximation.

In this work, we present an efficient approximation of the substrate integrated waveguide by cascading the unit cell structures based on segmentation technique, mode matching method and Floquet theorem.

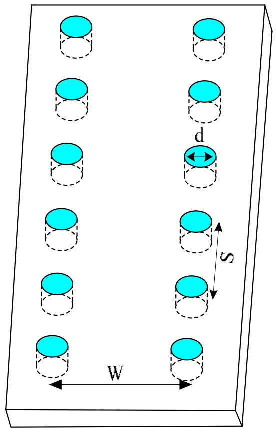

Fig. 1. Geometry of practical substrate integrated waveguide.

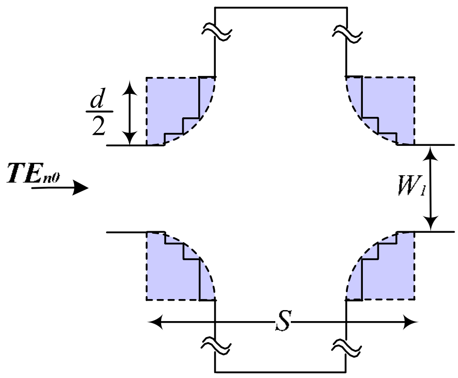

Fig. 2. A single unit cell structure.

To start with, we calculate the scattering parameters of the unit cell structure. By applying the Floquet's theorem on overall scattering matrix of a single unit cell, dispersion characteristic of the whole structure is obtained. On the other hand, an eigenvalue system is used to yield properties of the propagating modes in SIW.

The paper is organized as follows: In section II, the method of analysis is presented and mathematical formulation for a unit cell structure is described. Floquet's theorem is applied to acquire dispersion properties of the substrate integrated waveguide. Numerical results and comparison with other published results are presented in section III. Finally section IV concludes the paper.

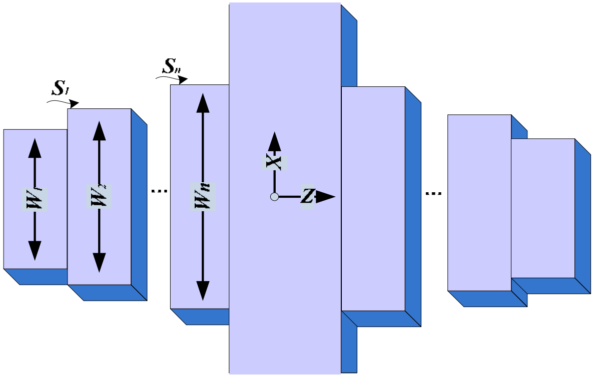

Fig. 3. The 3-D view of the single unit cell with cascaded rectangular waveguides.

II. Method of Analysis

The schematic view of the substrate integrated waveguide structure with its physical parameters are shown in Fig. 1. We take advantage of periodicity of the substrate integrated waveguide structure and investigate a unit cell for each period length as shown in Fig. 2.

In the unit cell, each single via is stair case segmented to cascading of rectangular waveguides with small widths. This approximation makes the analysis of substrate integrated waveguide more accurate and modifies the study to obtain the scattering parameters of the unit cell structure in comparison with [3]. As the unit cell is composed of consecutive waveguide sections, we calculate the scattering parameters of the junction between two waveguides with different cross-sections, as shown in Fig.3.

The electromagnetic field in first waveguide can be expressed as

$$E_t^{(1)}(x,y,z) = \sum_n V_n^{(1)}(z) e_n^{(1)}(x,y) \tag{1a}$$

$$H_t^{(1)}(x,y,z) = \sum_n I_n^{(1)}(z)\, h_n^{(1)}(x,y) \tag{1b}$$

And for the second waveguide,

$$E_t^{(2)}(x,y,z) = \sum_m V_m^{(2)}(z)\, e_m^{(2)}(x,y) \tag{2a}$$

$$H_t^{(2)}(x,y,z) = \sum_m I_m^{(2)}(z)\, h_m^{(2)}(x,y) \tag{2b}$$

At $z = 0$, the boundary condition imposes the continuity of the tangential electric and magnetic fields on the plane of the junction. Applying orthogonality of the modes, relations between the field expansion coefficients of the two waveguides are obtained as

$$V_m^{(2)} = \sum_n V_n^{(1)} \int_{S_i} e_m^{(2)} e_n^{(1)}\, dS \tag{3a}$$

$$I_n^{(1)} = \sum_m I_m^{(2)} \int_{S_i} e_m^{(2)} e_n^{(1)}\, dS \tag{3b}$$

Where the V's and I's are the equivalent voltages and currents evaluated at $z = 0$. The equations (3a) and (3b) can be simplified (shown) as follow

$$\begin{aligned} [V_2] &= [W]\,.\,[V_1] \\ [I_1] &= -[W]^T\,.\,[I_2] \end{aligned} \tag{4}$$

Where the matrix $[W]$ is as

$$w_{m,n} = \int_{S_i} e_m^{(2)} e_n^{(1)}\, dS = \int_{S_i} h_m^{(2)} h_n^{(1)}\, dS \tag{5}$$

Scattering parameters of the step junction can be obtained by expressing the voltages and currents in terms of incident and reflected waves [8]

$$\begin{aligned} [V_i] &= \left[g^{(i)}\right]^{-1}.\,([a_i] + [b_i]) \\ [I_i] &= \left[g^{(i)}\right]^{-1}.\,([a_i] - [b_i]) \end{aligned} \tag{6}$$

With $i = 1, 2$ and $\left[g^{(i)}\right] = \operatorname{diag}\left[\sqrt{Y_{m,n}^{(i)}}\right]$. By some mathematical manipulation, following scattering parameters are obtained

$$\begin{aligned} S_{11} &= [U] - [S_{12}].\left[g^{(2)}\right].[W].[g^{(1)}]^{-1} \\ S_{12} &= 2\left[g^{(1)}\right].[P].[W]^T.[g^{(2)}] \\ S_{21} &= [S_{12}]^T \\ S_{22} &= [S_{21}].\left[g^{(1)}\right]^{-1}.[W]^T.\left[g^{(2)}\right] - [U] \end{aligned} \tag{7}$$

In which $[U]$ is the unit matrix and $[P]$ is as follow

$$[P] = \left(\left[Y_c^{(1)}\right] + [W]^T.[Y_c^{(2)}]\ \ [W]\right)^{-1} \tag{8}$$

Where $[Y_c^{(i)}] = \operatorname{diag}\,[Y_{c,m}^{(i)}]$ and $[Y_{c,m}^{(i)}]$ is the characteristic admittances of the i^{th} waveguide.

In order to obtain scattering parameters of the unit cell structure, we cascade scattering parameters of consecutive step discontinuities.

Taking into account periodicity of the whole structure in z-direction, we employ Floquet's theorem as follows [9]

$$E(x,y,mSz) = E(x,y,z)e^{-mS\gamma z} \tag{9a}$$

$$H(x,y,mSz) = H(x,y,z)e^{-mS\gamma z} \tag{9b}$$

In which S, m and γ are respectively period length, an integer number and propagation constant. Propagation constant of the whole structure can be obtained as

$$\gamma = \frac{cos^{-1}(\frac{A+D}{2})}{S} \tag{10}$$

Where A and D are elements of transfer matrix of the unit cell obtained from scattering parameters.

III. Simulation Results and discussion

This method has been applied to the structure shown in Fig. 1 with parameters as follow

$W = 7.2$ mm

$S = 2$ mm

$h = 0.508$ mm

$d = 0.8$ mm

$\varepsilon_r = 2.33$

Fig. 4 shows dispersion diagram of TE_{10}-mode as a function of diameter of holes in comparison with other published results and proposed methods in [3] and [5]. As can be seen, propagation constant in our proposed method is almost the same as result in [5] and is in good agreement with result from empirical equation in [7]. Also comparing with [3], results from proposed method are more accurate. This gain is due to stair case segmentation which has been applied, causing better approximation in recent proposed method.

The cut off frequencies of the TE_{10} and TE_{20} modes with respect to the spacing W of the metalized holes is demonstrated in Fig. 5. Simulation results show a good agreement with the results in [10].

Fig. 6 illustrates the electric field in gap region. As can be seen, the electric field of the fundamental mode is mainly concentrated in the central portion of gap. This guarantees matching of the sections and consequently much small radiation from the gaps.

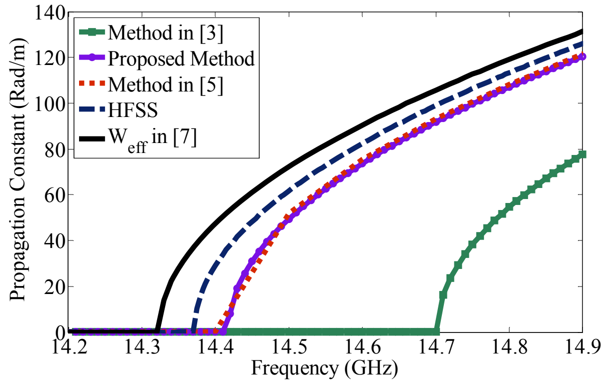

Fig.4 . Dispersion diagram of the approximated substrate integrated waveguide in comparison to results from [3], [5] and [7].

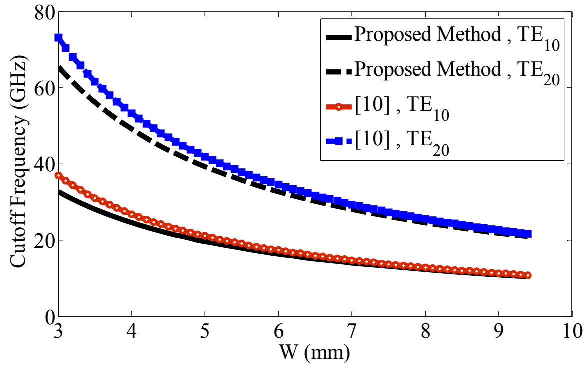

Fig. 5. Cutoff frequencies of the TE_{10} and TE_{20} modes of the approximated substrate integrated waveguide in proposed method comparing with [10] versus W.

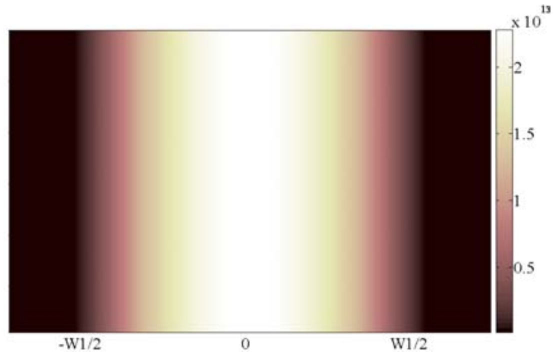

Fig. 6. Magnitude of the electric field of fundamental mode inside the unit cell.

IV. Conclusion

In this paper, an accurate analysis of the substrate integrated waveguide is presented using mode matching technique. Segmentation method is applied to via holes to approximate the SIW. A unit cell is considered in the whole periodic structure in order to obtain scattering parameters of the SIW. Dispersion characteristic and cutoff frequencies of the first two dominant modes of the approximated substrate integrated waveguide are acquired. Through comparison, good agreement between proposed method and other published results is obtained. Simplicity and fast computer simulation in addition to accuracy of the results are significant advantages of the proposed method.

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