# Analysis of Dispersion Characteristic of Substrate Integrated Waveguide Based on Mode Matching Method

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Abstract—In this paper,  $\beta$ - $\omega$  of substrate integrated waveguide is studied. First, a unit cell of the whole structure is analyzed by mode matching technique. Then by applying Floquet theory into the solution process, dispersion characteristic of the periodic structure is obtained and effects of various parameters of the structure on the response are described. Finally, proposed method is validated by comparing our results with other published results and Ansoft's HFSS.

Index Terms—Substrate Integrated Waveguide, Dispersion Characteristic, Mode Matching Technique and Floquet Theory.

### I. INTRODUCTION

In multi-layer microwave integrated circuits such as low temperature co-fired ceramics or multi-layer printed circuit boards, substrate integrated waveguide (SIW) is fabricated with sidewalls composing of periodic arrays of metallic via holes, largely preserving the advantages of both the conventional rectangular waveguide and microstrip [1], [2].

Active devices in the form of chips are often surface mounted on a planar carrier substrate while passive components such as a diplexer and filter are usually designed on the basis of rectangular waveguide or other non-planar technologies.

Using substrate integrated waveguide takes advantage of simply integrating passive and active components on a single substrate as a compact and high performance subsystem [3], [4]. This reduces size, weight, cost and greatly enhancing manufacturing reliability. Although the dispersion characteristic of a substrate integrated waveguide is similar to a waveguide, a little difference is considerable as a result of possible leakage from the periodic gaps [5], [6].

Many researches have been done on applications of substrate integrated waveguide in microwave circuits, but analysis of dispersion characteristics of substrate integrated waveguide is still developing [7], [8].

In this paper, mode matching technique is employed along with Floquet theorem to acquire  $\beta$ - $\omega$  diagram of the whole structure. For this purpose, the practical substrate integrated waveguide in Figure.1 is approximated with structure shown in Figure. 2 by considering the fact that pointing power toward gaps is rapidly evanesced. It makes this method of analysis more expediting in comparison with other full-wave numerical methods.

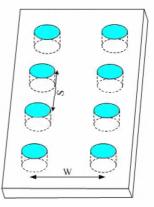


Fig. 1. Schematic view of practical substrate integrated waveguide.

The paper is organized as follows; In section II, the method of analysis is described and mathematical formulation for a unit cell is presented. Numerical results and comparison with other published results are presented in section III. Finally, section IV concludes the paper.

## **II. METHOD OF ANALYSIS**

The analysis of the substrate integrated waveguide structure is commenced with the expansion of eigen modes in the unit cell structure shown in Figure. 3, based on mode matching method technique. For describing the behavior of two junctions, we use potential function in (1) for each region. Considering  $TE_{10}$ - mode as an excitation mode, only  $TE_{n0}$ modes (*n* is odd) will exist in the structure. This, in turn, is based on the fact that no modes with field variation in the *y*-direction will be excited due to the topology.

$$\begin{aligned} \varphi_{1} &= \cos(\frac{\pi x}{W_{1}})e^{-j\beta_{1}^{(1)}z} + \sum_{n}A_{n}\cos(\frac{n\pi x}{W_{1}})e^{j\beta_{n}^{(1)}z} \\ \varphi_{2} &= \sum_{n}\left(B_{n}\cos(\frac{n\pi x}{W_{2}})e^{-j\beta_{n}^{(2)}z} + C_{n}\cos(\frac{n\pi x}{W_{2}})e^{j\beta_{n}^{(2)}z}\right) \\ \varphi_{3} &= \sum_{n}D_{n}\cos(\frac{n\pi x}{W_{1}})e^{-j\beta_{n}^{(1)}(z-a)} \end{aligned}$$
(1)

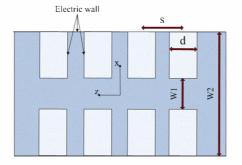


Fig. 2. Configuration of the substrate integrated waveguide in our proposed method.

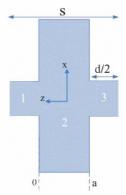


Fig. 3. Unit cell structure with period S and two junctions at z = 0 and z = a.

where,

$$\beta_n^{(1)} = \sqrt{\omega^2 \mu \epsilon - (\frac{n\pi x}{W_1})^2}$$
$$\beta_n^{(2)} = \sqrt{\omega^2 \mu \epsilon - (\frac{n\pi x}{W_2})^2}$$
(2)

In which  $A_n$ ,  $B_n$ ,  $C_n$  and  $D_n$  are the coefficients of the eigen vectors in unit cell. Applying the continuity of tangential electric and magnetic fields at z = 0 and z = a, the following formulations are obtained:

$$\begin{split} \vec{V_1} + \vec{M_1} \vec{A} &= \vec{N_1} \vec{B} + \vec{P_1} \vec{C} \\ \vec{V_2} + \vec{M_2} \vec{A} &= \vec{N_2} \vec{B} + \vec{P_2} \vec{C} \\ \vec{Q_1} \vec{A} &= \vec{N_3} \vec{B} + \vec{P_3} \vec{C} \\ \vec{Q_2} \vec{A} &= \vec{N_4} \vec{B} + \vec{P_4} \vec{C} \end{split}$$
(3)

where the elements of matrices  $\overline{M}$ ,  $\overline{N}$ ,  $\overline{P}$  and  $\overline{Q}$  are constant values calculated from the inner products of eigen modes in different regions. Thereby, scattering and transmitting components can be derived. Considering periodicity of the structure in z-direction, we employ Floquet theorem as follows:

$$\begin{split} \vec{E}(x,y,mSz) &= \vec{E}(x,y,z)e^{-mS\gamma z} \\ \vec{H}(x,y,mSz) &= \vec{H}(x,y,z)e^{-mS\gamma z} \end{split} \tag{4}$$

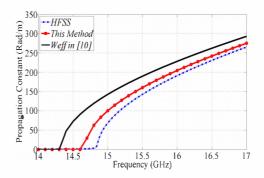


Fig. 4. Comparison between our proposed method and [10].

In which S, m and  $\gamma$  are respectively period length, an integer number and propagation constant. It can be seen that the phase factor is the eigen value of the ABCD matrix [9]:

$$\gamma = \frac{\cos^{-1}\left(\frac{(A+D)}{2}\right)}{S} \tag{5}$$

By calculating the ABCD elements of the unit cell from scattering parameters defined in (3), and utilizing (5), propagation constant of substrate integrated waveguide can be obtained.

# **III. NUMERICAL RESULTS**

According to Figure. 2 the geometric parameters of the structure are:

- $\epsilon_r = 2.33$
- S = 2 mm

• 
$$d = 0.8 \text{ mm}$$

• W = 7.2 mm

Figure. 4 indicates dispersion characteristic of the structure versus frequency of operation in comparison to results from [10] and HFSS with the same dimensions. It shows that our result is in good agreement with HFSS. Also the difference between our result and [10] is due to our approximation. Figure. 5 shows the effect of diameter of via holes on propagation constant of substrate integrated waveguide with holding S and W as a constant. When diameter is increased, dispersion characteristic of substrate integrated waveguide approaches to a rectangular waveguide's.

In Figure. 6 the behavior of analysis to changes in K, the ratio of  $W_2$  to  $W_1$ , is shown. It illustrates that this method accurately converges to the constant dispersion characteristic with increasing the K. As the K increased, the minimum number of modes which guaranties the convergence of method is described in Figure. 6 by n. This figure implies that K = 2 is a good choice to approach the proper accuracy, while K = 1 shows the dispersion characteristic of a rectangular waveguide. Based on results, the most significant advantage of our method is its good convergence and short time of execution.

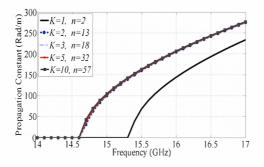


Fig. 5. Dispersion characteristic of substrate integrated waveguide as a function of diameter of via holes, with S=2 mm, W=7.2 mm and  $\epsilon_r=2.33$ .

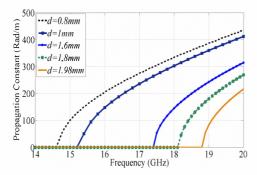


Fig. 6. Dispersion characteristic of substrate integrated waveguide as a function of K in proposed structure with  $W_2 = KW_1$ , S = 2 mm, W = 7.2 mm and  $\epsilon_r = 2.33$ .

## **IV. CONCLUSIONS**

In this paper, the mode matching method has been used for analyzing of substrate integrated waveguide. By approximating a practical substrate integrated waveguide with cascaded unit cells, a simple method for analyzing of substrate integrated waveguide is presented. Dispersion characteristic is obtained by applying Floquet theory on the scattering parameters of a unit cell. The effect of dimensions and the diameter of via holes in propagation constant are studied. Numerical results have been presented in comparison with other works and Ansoft's HFSS. The results show that our proposed method is a simple and accurate technique to characterizing the substrate integrated waveguide.

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